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Dynamic Event-Triggering Resilient Consensus for Heterogeneous Multi-Agent Systems (MASs) Against Malicious Attacks

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1. Research Background and Significance

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➢ Research Background

Application of MASs in Various Fields

Swarm Performance

Agriculture

Cooperative Detection

Target Tracking

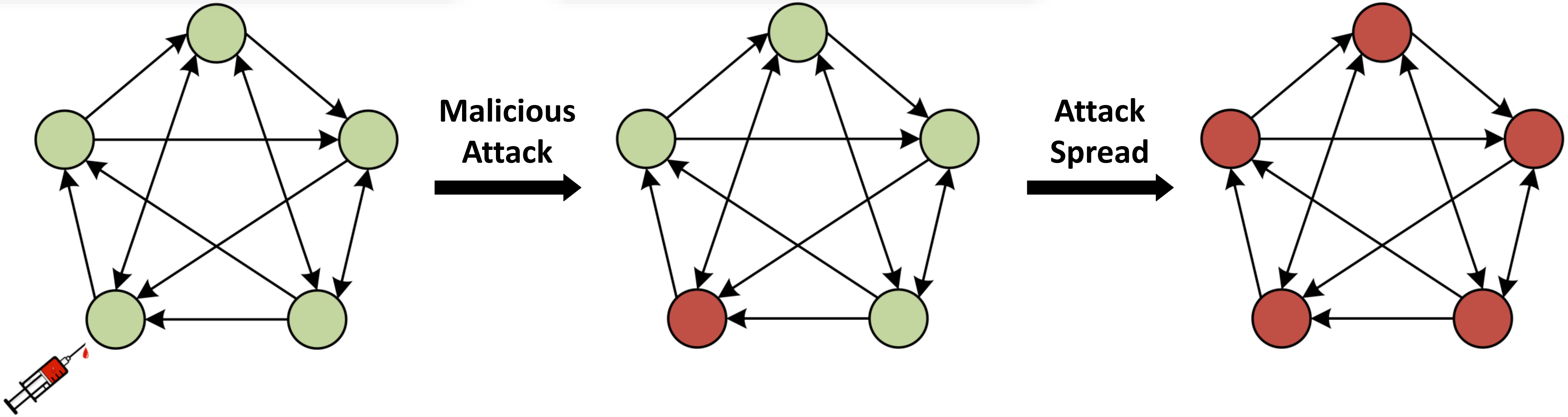
1. Research Background and Significance

➢ Research Significance

The swarm operates normally

Some drones are under attack

Swarm collapse



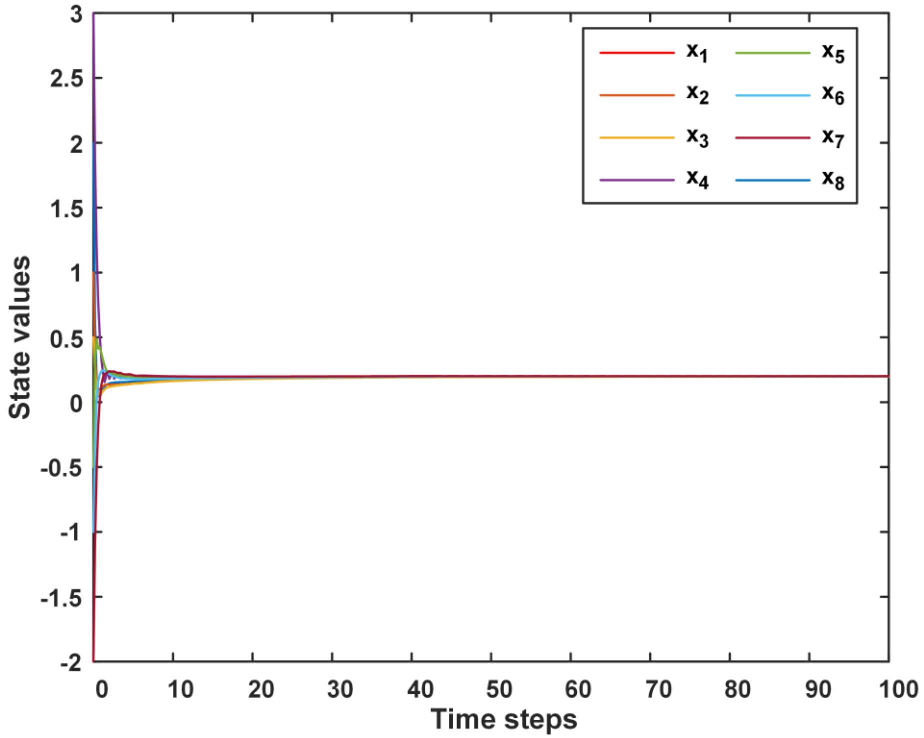
Research Significance

- Consider a single-integrator MAS modelled by

$$x_i[k+1]=x_i[k]+u_i[k]$$

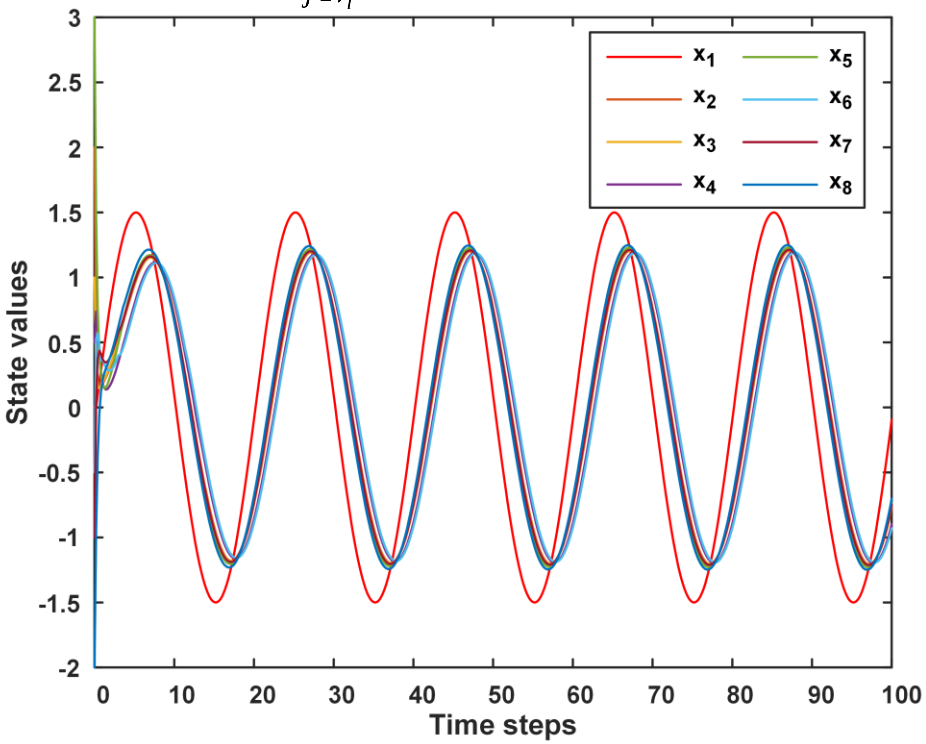
- A consensus-seeking protocol[1] is introduced as

$$u_i[k]=\sum_{j\in\mathcal{V}_i} a_{ij}[k]\big(x_j[k]-x_i[k]\big)$$



Without Malicious attack

Malicious Attack



Agent 1 is subject to malicious attack

Motivating Question: How to address resilient consensus problem when the MAS is under attack?

[1] Ren W, Cao Y. Distributed coordination of multi-agent networks: emergent problems, models, and issues. London: Springer, 2011.

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2. Literature Review

➢ Review of Resilient Control Methods for MASs

Attack Detection and Isolation Method[2,3]	Attack Tolerant Method[4,5]
Detecting and isolating malicious agents	Deleting malicious state values
Agents are equipped with a bank of observers	More lightweight with less computational complexity
Massive data processing	Less data processing
High network connectivity required	Less prior information required
Unknown input observers	**Mean Subsequent reduced (MSR) algorithm**

[2] I. Shames, A. M. Teixeira, H. Sandberg, and K. H. Johansson, "Distributed fault detection for interconnected second-order systems," *Automatica*, vol. 47, no. 12, pp. 2757–2764, 2011.

[3] Z. Tang, M. Kuijper, M. S. Chong, I. Mareels, and C. Leckie, "Linear system security-detection and correction of adversarial sensor attacks in the noise-free case," *Automatica*, vol. 101, pp. 53–59, 2019.

[4] H. J. LeBlanc, H. Zhang, X. Koutsoukos, and S. Sundaram, "Resilient asymptotic consensus in robust networks," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 4, pp. 766–781, 2013.

[5] Y. Zhai, Z. Liu, M. Ge, G. Wen, X. Yu, and Y. Qin, "Trusted-region subsequence reduction for designing resilient consensus algorithms," *IEEE Transactions on Network Science and Engineering*, vol. 8, no. 1, pp. 259–268, 2020.

2. Literature Review

➢ Existing Resilient Control Methods based on MSR

Literature	Year	System Model	Communication Mode	Objective
[4]	2013	Single-integrator MASs	Time-triggered	Exact resilient consensus
[6]	2017	Double-integrator MASs	Time-triggered	Exact resilient consensus
[7]	2022	General linear MASs	Time-triggered	Exact resilient consensus
[8]	2020	Single-integrator MASs	Static event-triggered	Bounded resilient consensus
[9]	2021	Single-integrator MASs	Static event-triggered	Bounded resilient consensus
Our research	2024	**Heterogeneous MASs**	**Dynamic event-triggered**	**Exact resilient consensus**

[4] H. J. LeBlanc, H. Zhang, X. Koutsoukos, and S. Sundaram, "Resilient asymptotic consensus in robust networks," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 4, pp. 766–781, 2013.
[6] S. M. Dibaji and H. Ishii, "Resilient consensus of second-order agent networks: Asynchronous update rules with delays," *Automatica*, vol. 81, pp. 123–132, 2017.
[7] Y. Bai and J. Wang, "Resilient consensus of continuous-time linear networked systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 69, no. 8, pp. 3500–3504, 2022.
[8] Y. Wang and H. Ishii, "Resilient consensus through event-based communication," *IEEE Transactions on Control of Network Systems*, vol. 7, no. 1, pp. 471–482, 2020.
[9] N. Wang, and Y. Liu, "Resilient consensus of second-order multi-agent systems subject to malicious attacks," in *Proc. of the 2021 Chinese Intelligent Systems Conference*. Springer, 2021, pp. 10-21.

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➢ Dynamic Modelling for Heterogeneous MASs

- Consider a heterogeneous MASs consisting of **first-order and second-order agents**.
- The **discrete system dynamics** are mathematically expressed as follows:

◆ **Dynamic equations:**

$$\dot{x}_i(t)=u_i(t), \qquad i\in\mathcal{N}_1$$

$$\begin{cases}\dot{x}_i(t)=v_i(t)\\ \dot{v}_i(t)=u_i(t)\end{cases}, \qquad i\in\mathcal{N}_2$$

Sampling Period T → **Discretization**

$$x_i[(k+1)T]=x_i[kT]+Tu_i[kT], \qquad i\in\mathcal{N}_1$$

$$\begin{cases}x_i[(k+1)T]=x_i[kT]+Tv_i[kT]+\dfrac{T^2}{2}u_i[kT]\\ v_i[(k+1)T]=v_i[kT]+Tu_i[kT]\end{cases}, \qquad i\in\mathcal{N}_2$$

↓ **Replace kT with k**

$$x_i[k+1]=x_i[k]+Tu_i[k], \qquad i\in\mathcal{N}_1$$

$$\begin{cases}x_i[k+1]=x_i[k]+Tv_i[k]+\dfrac{T^2}{2}u_i[k]\\ v_i[k+1]=v_i[k]+Tu_i[k]\end{cases}, \qquad i\in\mathcal{N}_2$$

◆ **Symbol meaning:**

$x_i(t)\in\mathbb{R}$: **Position of agent *i* at time *t***

$v_i(t)\in\mathbb{R}$: **Velocity of agent *i* at time *t***

$u_i(t)\in\mathbb{R}$: **Input of agent *i* at time *t***

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➢ Problem Statement

The MAS is said to achieve **exact resilient consensus** if the following two conditions hold for any initial state values of agents:

◆ **Resilience:** For each normal agent, it holds $x_i[k] \in \mathcal{S}, \quad \forall k \in \mathbb{Z}_+$, where $\mathcal{S}$ denotes the safety interval.

◆ **Exact Consensus:** For each normal agent, it holds $\lim_{k\to\infty} |x_i[k] - x_j[k]| = 0, \quad \forall i, j \in \mathcal{N}$.
For each normal second-order agent, it holds $\lim_{k\to\infty} v_l[k] = 0, \forall l \in \mathcal{N}_2$.

Research Question: How to design control protocols to protect the heterogeneous MAS from malicious attacks and achieve resilient consensus?

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Control Protocol Design

- For normal first-order agents, the control protocol relies on the **difference** between neighbor's positions and their own positions.
- For normal first-order agents, the control protocol considers both the **difference** and their own **velocities**.

Control protocol for heterogeneous MAS

$$u_i[k]=\begin{cases}\gamma\sum\limits_{j\in\mathcal{R}_i[k]}a_{ij}[k]\left(x_j[k]-x_i[k]\right), & i\in\mathcal{N}_1\\ \alpha\sum\limits_{j\in\mathcal{R}_i[k]}a_{ij}[k]\left(x_j[k]-x_i[k]\right)-\beta v_i[k], & i\in\mathcal{N}_2\end{cases}$$

Compared with the control protocol in [1]

$$u_i[k]=\sum_{\boxed{j\in\mathcal{V}_i}}a_{ij}[k]\left(x_j[k]-x_i[k]\right) \quad\Rightarrow\quad u_i[k]=\boxed{\gamma}\sum_{\boxed{j\in\mathcal{R}_i[k]}}a_{ij}[k]\left(x_j[k]-x_i[k]\right)$$

Control gain
regulate the system state more accurately

Neighbor set
Do not use all neighbor positions for update

[1] Ren W, Cao Y. *Distributed coordination of multi-agent networks: emergent problems, models, and issues.* London: Springer, 2011.

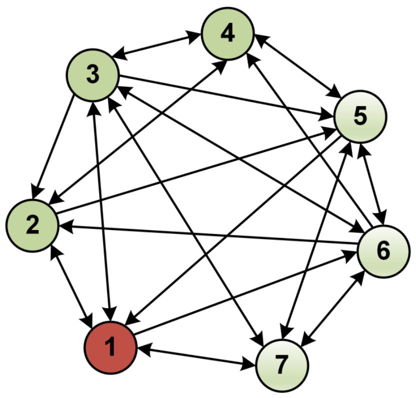
MSR Algorithm

1. Sorting

2.Deleting

3.Updating

Example：



$$[x_1[0],\cdots,x_7[0]]=[10,8,6,7,4,-5,6]$$

$$[v_5[0],v_6[0],v_7[0]]=[3,-2,-1],\quad f=2$$

Agent 2: $x_2[0]=8,\ \ x_j[0]=[10,6,7,-5]$

1. Sorting

$$S=[-5,6,7,8,10]^{\mathrm{T}}$$

2. Deleting

$$S_{del}=[\cancel{-5},\cancel{6},7,8,\cancel{10}]^{\mathrm{T}},\quad \mathcal{R}_i[0]=\{4\}$$

3. Updating

$$x_i[1]=x_i[0]+\sum_{\boxed{j\in\mathcal{R}_i[0]}} a_{ij}[0]\left(x_j[0]-x_i[0]\right)$$

3. Methodology

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MSR Algorithm

1. Sorting
2. Deleting
3. Updating

Require $x_j[k]$ at each time step

Heavy communication load

Event-based MSR (E-MSR) Algorithm[8]

1. Sorting
2. Deleting
3. Updating
4. Checking

[8] Y. Wang and H. Ishii, "Resilient consensus through event-based communication," *IEEE Transactions on Control of Network Systems*, vol. 7, no. 1, pp. 471–482, 2020.

➢ Static Event-Triggered Mechanism[8]

- The control protocol is also modified as

$$u_i[k]=\begin{cases}\gamma\sum_{j\in\mathcal{R}_i[k]} a_{ij}[k]\big(\hat{x}_j[k]-x_i[k]\big), & i\in\mathcal{N}_1\\ \alpha\sum_{j\in\mathcal{R}_i[k]} a_{ij}[k]\big(\hat{x}_j[k]-x_i[k]\big)-\beta v_i[k], & i\in\mathcal{N}_2\end{cases}$$

Auxiliary variable

- The update of auxiliary variable depends on

$$\hat{x}_i[k+1]=\begin{cases}x_i[k+1], & \text{if}\quad f_i[k]>0,\\ \hat{x}_i[k], & \text{otherwise.}\end{cases}$$

- The static event-triggered function (SETF) is designed as

$$f_i[k]=\underbrace{\big|x_i[k+1]-\hat{x}_i[k]\big|}_{\text{Error term } |e_i[k]|}-\underbrace{\left(c_0+c_1e^{-\alpha k}\right)}_{\text{Threshold}}$$

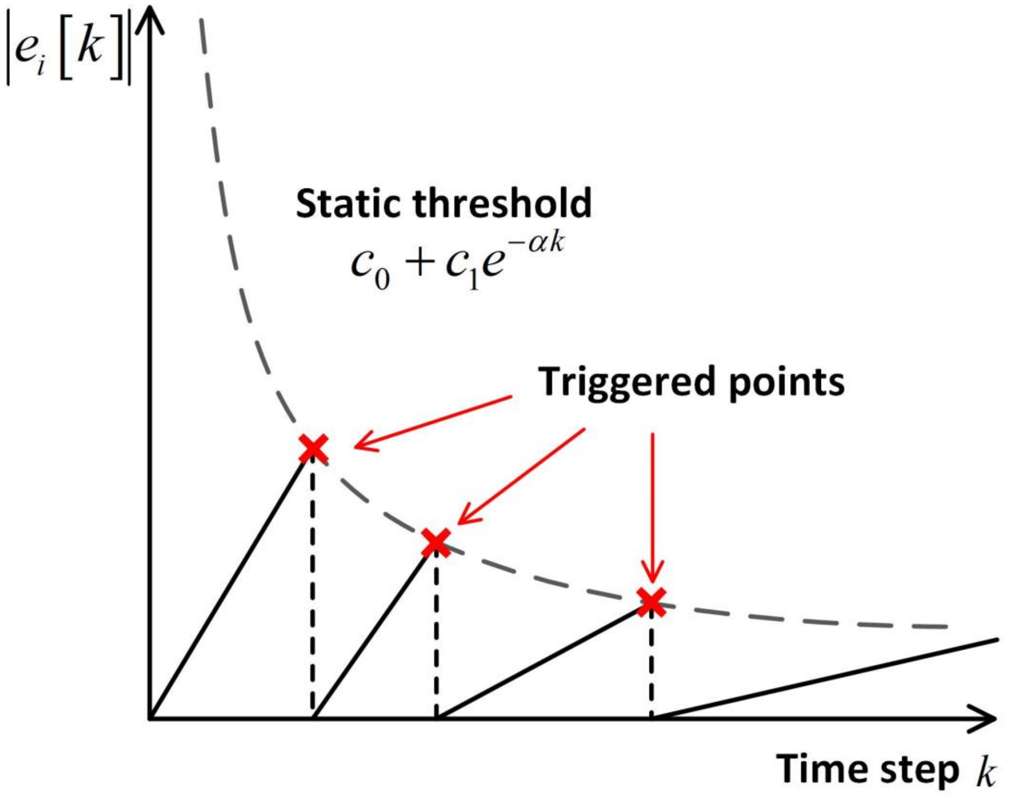

Illustration of the static event-triggered mechanism.

[8] Y. Wang and H. Ishii, “Resilient consensus through event-based communication,” *IEEE Transactions on Control of Network Systems*, vol. 7, no. 1, pp. 471–482, 2020.

E-MSR Algorithm

1. Sorting

2. Deleting

3. Updating

4.Checking

Step 4: Agent i checks whether SETF triggers or not and sets $\hat{x}_i[k+1]$ as

$$\hat{x}_i[k+1]=\begin{cases} x_i[k+1], & \text{if SETF triggers,} \\ \hat{x}_i[k], & \text{otherwise.} \end{cases}$$

Shortcomings:

- Only bounded resilient consensus can be achieved due to the existence of constant term c_0, i.e.,

$$\lim_{k\to\infty}\left|x_i[k]-x_j[k]\right|\leq c, \quad \forall k\in\mathbb{Z}_{\geq 0}.$$

- The threshold cannot dynamically adjust as the error term changes.

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➢ Dynamic Event-Triggering Mechanism[10]

- The control protocol is also modified as

$$u_i[k]=\begin{cases}\gamma\sum_{j\in\mathcal{R}_i[k]}a_{ij}[k]\left(\hat{x}_j[k]-x_i[k]\right), & i\in\mathcal{N}_1\\ \alpha\sum_{j\in\mathcal{R}_i[k]}a_{ij}[k]\left(\hat{x}_j[k]-x_i[k]\right)-\beta v_i[k], & i\in\mathcal{N}_2\end{cases}$$

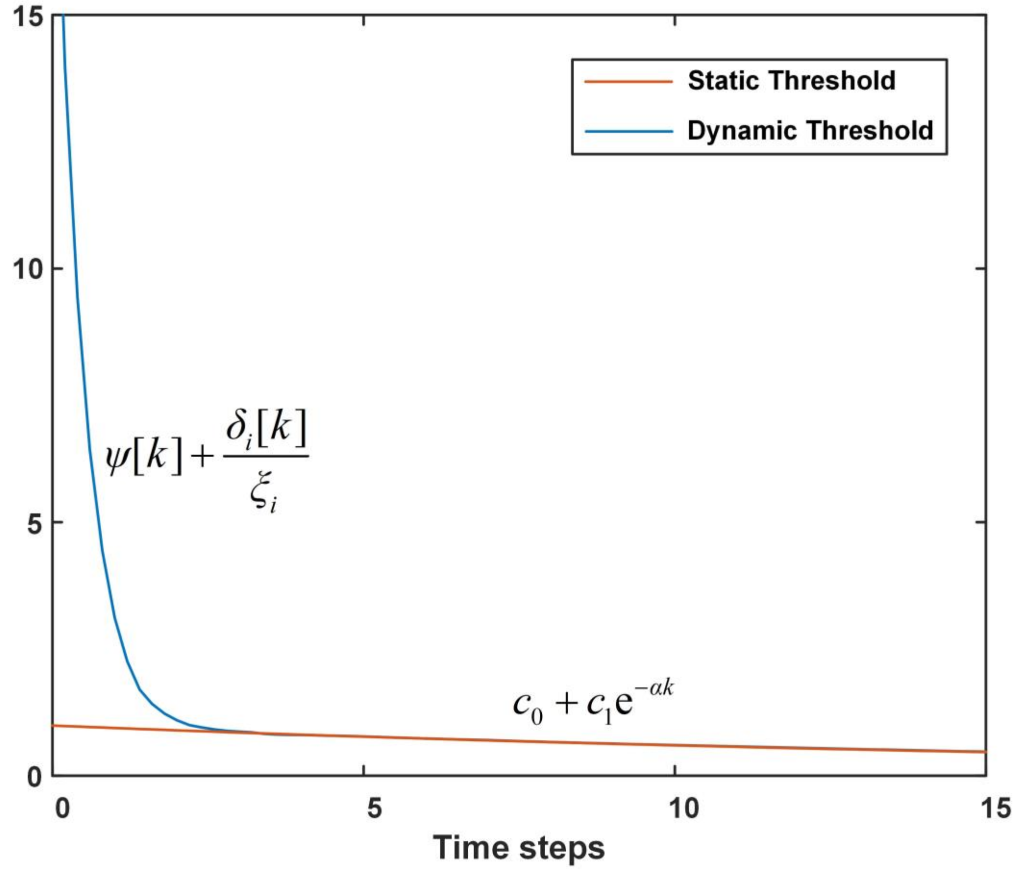
- A dynamic variable $\delta_i[k]$ is introduced, whose state update follow

$$\delta_i[k+1]=(1-\theta_i)\delta_i[k]+\eta_i(\psi[k]-|e_i[k]|)$$

Always greater than zero and exponentially converges to zero.

- The dynamic event-triggered function (DETF) is designed as

$$f_i[k]=|e_i[k]|-\left(\psi[k]+\frac{\delta_i[k]}{\xi_i}\right)$$



Comparison of the two triggering thresholds.

[10] Z. Liao, S. Wang, J. Shi, Y. Zhang, and Z. Sun, "Resilient consensus through dynamic event-triggered mechanism," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 71, no. 7, pp. 3463–3467, 2024.

Dynamic Event-triggered Mean Subsequence Reduced (DE-MSR) Algorithm[10]

1. Sorting

2. Deleting

3&5. Updating

4.Checking

Step 4: Agent i checks whether DETF triggers or not and sets $\hat{x}_i[k+1]$ as

$$\hat{x}_i[k+1] = \begin{cases} x_i[k+1], & \text{if SETF triggers}, \\ \hat{x}_i[k], & \text{otherwise}. \end{cases}$$

Step 5: Agent i updates its interval dynamic variable according to

$$\delta_i[k+1] = (1-\theta_i)\delta_i[k] + \eta_i(\psi[k] - |e_i[k]|)$$

[10] Z. Liao, S. Wang, J. Shi, Y. Zhang, and Z. Sun, "Resilient consensus through dynamic event-triggered mechanism," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 71, no. 7, pp. 3463–3467, 2024.

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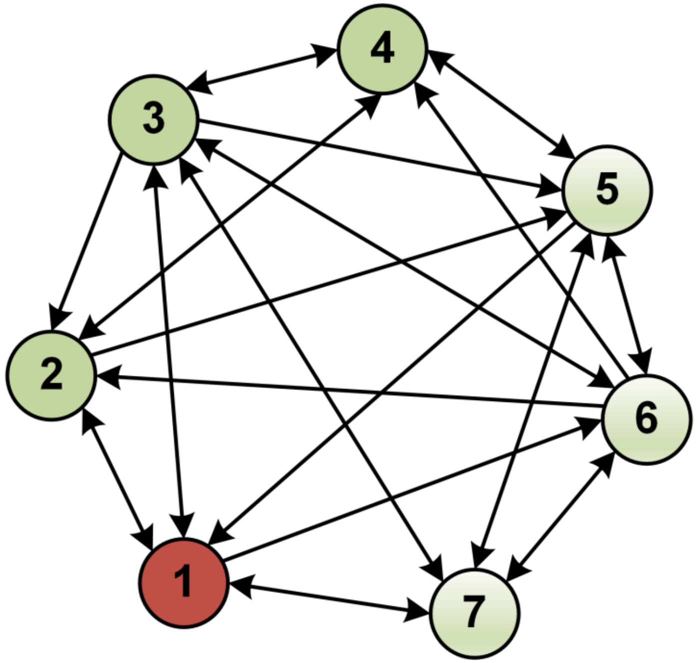
➢ A Comparative Numerical Example Between SETF and DETF

◆ Communication Topology



◆ Simulation Setting

- Agents 1,2,3,4: first-order; Agents 5,6,7: second-order
- Agent 1 is a **malicious agent**, whose motion follows

$$x_1[k] = 5 \times \sin(0.2 \times k) + 5$$

- **Initial state values** of all agents are denoted as

$$X[0] = [7,7,6,8,\text{-}5,8,6],\ V[0] = [0,0,0,0,3,-2,-1]$$

- SETF related parameters:

$$c_0 = 0.005,\ c_1 = 0.01,\ \alpha = 0.01$$

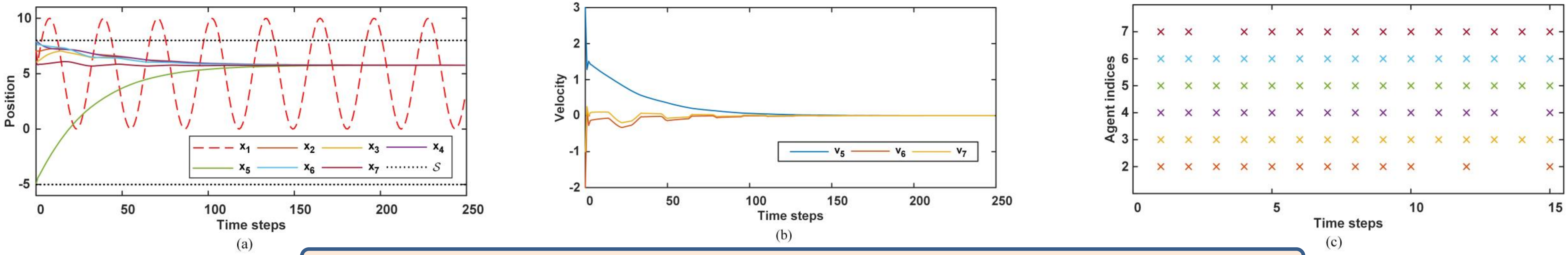
- DETF related parameters:

$$\xi_i = 10,\ \eta_i = 0.6,\ \theta_i = 0.35,\ \delta_i[0] = 15,\ \psi[k] = \mathrm{e}^{-\beta k},\ \beta = 0.01$$

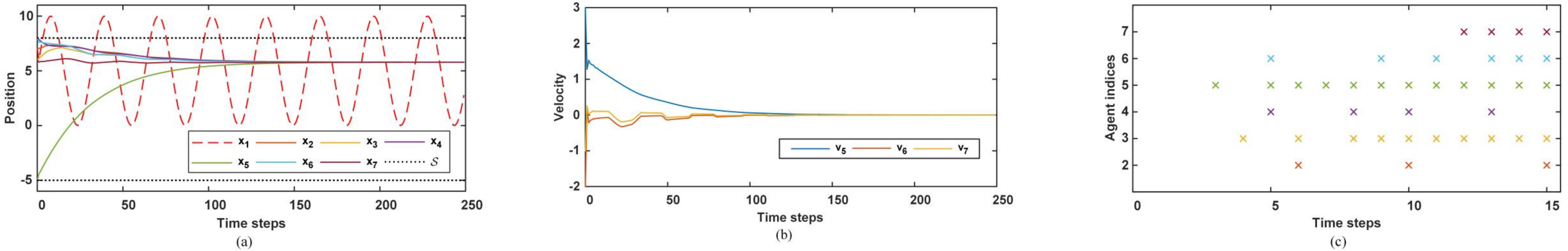
◆ Purpose

- To validate the **effectiveness** of the proposed DE-MSR algorithm.
- To show the **superiority** of DETF compared with SETF.

Simulation Results



Simulation Results with SETF: (a) Position; (b) Velocity; (c) Triggering behavior.



Simulation Results with DETF: (a) Position; (b) Velocity; (c) Triggering behavior.

Event-triggering mechanism	Event counts (within 250 time steps)					
	ag.2	ag.3	ag.4	ag.5	ag.6	ag.7
SETF	150	127	149	178	113	102
DETF	140	122	138	174	106	90

- The SETF and DETF show a similar convergence rate.
- The DETF is superior in reducing communication times.

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5. Conclusion and Future Work

➢ Conclusion

- Study the resilient consensus problem for **heterogeneous MASs**.
- Propose the **DE-MSR algorithm** to reduce communication overheads.
- Validate the **effectiveness** of the proposed method through numerical simulations.

➢ Prospection

Future work includes：

- MASs with **disturbances** generated from the external environment.
- More **general and complex tasks** (e.g., resilient distributed optimization).

Thank you so much for your attention!

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Full Paper **WeChat**