Dynamic Event-Triggering Resilient Coordination for Time-Varying Heterogeneous Networks

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***Abstract*—This study addresses the resilient coordination problem for heterogeneous multi-agent systems (MASs) consisting of first-order and second-order agents in time-invariant and time-varying networks. An internal dynamic variable is introduced to flexibly adjust the triggering threshold and facilitate the dynamic event-triggering condition (DETC). Under adversarial attacks, a novel resilient consensus strategy called *heterogeneous dynamic event-triggering mean-subsequence-reduced (HDE-MSR) algorithm* is further developed, which ensures that the positions of all healthy agents achieve consensus on the identical value and the velocities of all healthy second-order agents asymptotically approach zero despite the influence of faulty agents. Moreover, the resilient consensus in time-varying networks is further guaranteed by the introduction of jointly robust graphs. Finally, three case studies are provided to validate the effectiveness and superior performance of the HDE-MSR algorithm.**

***Index Terms*—Heterogeneous MAS, resilient coordination, dynamic event-triggering mechanism, time-varying network.**

I. Introduction

In recent years, much attention has been devoted to the investigation of multi-agent coordination problems under adversarial environments [1]–[5]. The openness of the network medium makes the multi-agent system (MAS) vulnerable to external malicious attacks, while the distributed characteristic makes the MAS difficult to identify and eliminate every potential attack. These attacks may influence the consensus state of the system through a single faulty agent, which would result in an overall system collapse. Traditional consensus-seeking methods are unable to ensure resilience against potential misbehavior. Consequently, the consensus issue under adversarial attacks, i.e., resilient consensus, has gained increasing attention in recent years [6]–[10]. It aims to guarantee that the state values of healthy agents converge to the identical value or an error range, despite the misbehavior of some faulty agents in the network.

Motivated by concern for system security, various techniques have been developed to mitigate or eliminate the effect of adversarial attacks. Generally, there are two categories of methods for addressing resilient consensus problems. One category is based on the idea of fault identification and isolation, which enables that faulty agents in the network to be first identified and then isolated [11], [12]. For distributed MASs, however, identifying and isolating faulty agents is challenging since these operations require agents to handle a massive amount of information. It is also impractical to identify all potential threats in large-scale distributed MASs.

The second category comprises a set of algorithms known as *Mean-Subsequence-Reduced* (MSR), which has garnered significant interest over the past decade. In the seminal study [13], the authors developed a weighted MSR algorithm (W-MSR) to ensure resilient consensus against Byzantine attacks. By implementing the W-MSR algorithm, healthy agents can ignore the information provided by potentially faulty agents or those with suspicious values, regardless of whether they are genuinely faulty. The identity of faulty agents remains undisclosed to healthy agents. Compared to fault detection and isolation strategies, MSR-based algorithms are more lightweight with less computational complexity. Agents in the network can also process less information and do not need to know the global network topology. Therefore, several studies have been devoted to addressing resilient consensus problems by adopting the idea of MSR in recent years [14]–[16].

Currently, the research on resilient consensus mainly concerns homogeneous MASs, i.e., agents in the network possess identical dynamic models. Typical examples include studies [17]–[19], which pursue resilient consensus for MASs with single-integrator dynamics. The paper [17] proposed a resilient consensus strategy for double-integrator MASs when the network is subject to adversarial attacks and communication delays. In [18], the leader-follower resilient consensus problem was addressed for MASs with first-order dynamics, where the consensus value can be arbitrarily designated by the leader agents. In [19], the authors extended resilient consensus to distributed optimization under adversarial environments, where each agent possesses a local cost function and the same single-integrator dynamics. Furthermore, the papers [20]–[22] pursue resilient consensus for MASs with double-integrator dynamics, which are frequently used models for autonomous mobile robots. To mention a few, the paper [20] developed

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a double-integrator position-based MSR (DP-MSR) algorithm to handle the resilient consensus problem for double-integrator MASs, when the network is subject to adversarial attacks and communication delays. In [21], a novel resilient coordination scheme was proposed to handle the cooperative source seeking problem for double-integrator multi-robot systems. Nevertheless, agents in the network may not have the same dynamics due to miscellaneous applications in practical scenarios, e.g., air-ground coordination. This fact motivates us to investigate resilient consensus problems for heterogeneous MASs.

Despite the widespread use of homogeneous MASs in resilient consensus problems, heterogeneity is still a crucial aspect for MASs. The paper [23] studied the resilient consensus problem for heterogeneous MASs composed of first-order and second-order agents in switching topologies. This heterogeneous system model was subsequently adopted in [24] to guarantee resilient consensus with bounded communication delays. In [25], the heterogeneous MAS consisting of first-order and second-order agents was also used and a static event-triggering mechanism was designed to achieve resilient consensus with reduced communication overheads. Considering that agents in the network may not have the same dynamics due to miscellaneous applications in practical scenarios, e.g., air-ground coordination and power transmission network, this paper will also investigate the resilient consensus problem for heterogeneous MASs consisting of first-order and second-order integrator dynamics. Furthermore, compared to [25], we not only introduce an internal dynamic variable and design a dynamic event-triggering mechanism, which further reduces communication times between agents and saves more communication resources, but also incorporate the dynamic event-triggering mechanism and the switching network topology simultaneously.

In most existing MSR-based resilient strategies, each healthy agent needs to communicate with its out-neighbors and in-neighbors at each time step to send its own states and receive its in-neighbors' states. This operation will cause considerable resource consumption and is unfavorable when communication resources are limited. To mitigate the heavy communication burden, the work [26] developed an event-based MSR (E-MSR) algorithm to defend against malicious attacks with reduced communication overheads. The designed event-triggering mechanism allows agents to transmit information only when necessary, specifically when a significant change of values occurs since the last transmission time. In some situations, agents only transmit information to their neighbors occasionally, thereby remarkably reducing the communication frequency. Motivated by this seminal study, the paper [27] adopted the E-MSR algorithm to ensure resilient consensus in multi-dimensional space. In [25], the E-MSR algorithm was employed to guarantee resilient consensus for heterogeneous MASs. However, these excellent studies only pursue a bounded consensus, which ensures that the positions of healthy agents converge to an error level. Moreover, the preset event functions are usually static and may not be adaptable to dynamic changes in agents' states. To overcome the above challenges, the dynamic event-triggering mechanism [28]–[30] will be introduced in this study to further mitigate the communication overheads and achieve exact resilient consensus.

In addition to heavy communication overheads, another concern regarding resilient consensus is varying network topologies, which may result from restricted communication capability or physical barriers during transmission. Compared with time-invariant networks, time-varying networks are more common in practice due to non-stationary environments and dynamic system behaviors. In the context of resilient control, the consideration of time-varying networks not only improves the practical viability of resilient algorithms but also relaxes the requirement that the network satisfies specific graph conditions at each time step. The varying network topology is also a reflection of system randomness [31]. Currently, there are only a few studies that have investigated resilient consensus problems in time-varying networks [23], [32], [33]. In [32], the notion of the time window was introduced and a sliding-window MSR (SW-MSR) algorithm was developed to guarantee resilience for networks of agents in time-varying graphs. With a comprehensive consideration of trust nodes and time windows, the paper [23] achieved resilient consensus for heterogeneous MASs over time-varying digraphs. In [33], the notion of jointly robust graph was introduced, and the necessary and sufficient conditions were provided to ensure resilient consensus for single-integrator and double-integrator MASs in switching digraphs. Although these promising studies enable agents in the network to sparsely connect with other agents at each time step, they still require that agents interact with others frequently to access their state values, which is unfavorable under limited communication resource conditions. In other words, how to guarantee resilient consensus in time-varying networks with limited communication resources still remains an open problem. Thus, the comprehensive consideration of event-triggering mechanisms and time-varying networks ought to be emphasized.

Motivated by the aforementioned observations, this study addresses the resilient consensus problem for heterogeneous MASs in time-invariant and time-varying networks and devises a dynamic event-triggering controller to guarantee resilient consensus with reduced communication overheads. The challenge of the research problem stems from the presence of faulty agents, heterogeneous system dynamics, the evolving dynamic variable, and the switching network topology. The research emphases of this work are highlighted as follows:

1) Compared to [25], [34] that addressed the resilient consensus problem for MASs using the event-triggering mechanism, we introduce an internal dynamic variable and design a dynamic event-triggering mechanism, which further reduces communication times between agents and saves more communication resources. Moreover, we discover that the positions of healthy agents are convex combinations of their previous positions and deduce the convex combination forms of the heterogeneous MAS dynamics.
2) To achieve resilient consensus with reduced communication overheads, a heterogeneous dynamic event-triggering MSR (HDE-MSR) algorithm is developed in this paper. Compared to the resilient consensus study

[35] based on the dynamic event-triggering mechanism, we extend the results to heterogeneous MASs to handle more complex system dynamics. Moreover, compared to the event-based resilient schemes [26], [27], the proposed HDE-MSR algorithm balances convergence performance as well as triggering performance and achieves exact resilient consensus.

3) The changes of dynamic variable and network topology at each time step make it difficult to analyze the stability of heterogeneous MASs. To address this challenge, we apply the proposed HDE-MSR algorithm to guarantee heterogeneous resilient consensus in switching topologies. The introduction of jointly robust graphs enables agents in the network to connect sparsely with other agents, thereby relaxing the requirement that the digraph should fulfill the graph robustness conditions at each time step. Verifiable sufficient conditions to ensure resilient consensus are further derived through a rigorous Lyapunov-function-based approach.

The rest of this study is organized as follows. Section II introduces some necessary graph theory notions, presents the dynamic event-triggering mechanism, and formulates the resilient consensus problem. Subsequently, the HDE-MSR algorithm and theoretical analysis are presented in Section III. Section IV illustrates the effectiveness and superiority of the theoretical results through three case studies. The conclusion and future research directions are given in Section V.

II. Problem Setup

A. Graph Theory Notions

Consider a heterogeneous MAS modelled by a time-invariant digraph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}=\{1,\dots,n\}$ is the node set and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the directed edge set. The edge $(j,i)\in\mathcal{E}$ indicates that agent i has access to information of agent j. The sets of in-neighbors and out-neighbors for agent i are defined as $\mathcal{N}_i^+=\{j\in\mathcal{V}|(j,i)\in\mathcal{E}\}$ and $\mathcal{N}_i^-=\{j\in\mathcal{V}|(i,j)\in\mathcal{E}\}$, respectively.

We consider the situation that the node set $\mathcal{V}$ contains adversarial agents. Therefore, two essential notions for time-invariant digraphs are given to describe the resilience, i.e., reachability and robustness.

Definition 1 (r-reachable set [13]): Consider a time-invariant digraph $\mathcal{G}=(\mathcal{V},\mathcal{E})$ and a nonempty subset $\mathcal{S}\subset\mathcal{V}$. $\mathcal{S}$ is said to be r-reachable if $\exists\ i\in\mathcal{S}$ such that $|\mathcal{N}_i^+\setminus\mathcal{S}|\geq r$, where $r\in\mathbb{Z}_+$.

Definition 2 (r-robust graph [13]): A time-invariant digraph $\mathcal{G}=(\mathcal{V},\mathcal{E})$ is said to be r-robust if for each pair of nonempty and disjoint subsets $\mathcal{S}_1,\mathcal{S}_2\subset\mathcal{V}$, at least one of $\mathcal{S}_1$ or $\mathcal{S}_2$ is r-reachable, where $r\in\mathbb{Z}_+$.

Regarding the time-varying digraph $\mathcal{G}[k]=(\mathcal{V},\mathcal{E}[k])$, the corresponding versions of reachable set and robust graph are defined below.

Definition 3 (Jointly r-reachable set [33]): Consider a time-varying digraph $\mathcal{G}[k]=(\mathcal{V},\mathcal{E}[k])$ and a nonempty subset $\mathcal{S}\subset\mathcal{V}$. $\mathcal{S}$ is said to be jointly r-reachable if there exists an infinite sequence of uniformly bounded and non-overlapping time intervals $[k_h,k_{h+1})$ such that in each time interval $[k_h,k_{h+1})$, there exist a time step $k_j\in[k_h,k_{h+1})$ and an agent $i_j\in\mathcal{S}$ such that $|\mathcal{N}_{i_j}^+[k_j]\backslash\mathcal{S}|\geq r$, where $r\in\mathbb{Z}_+$.

Definition 4 (Jointly r-robust graph [33]): A time-varying digraph $\mathcal{G}[k]=(\mathcal{V},\mathcal{E}[k])$ is said to be jointly r-robust if for each pair of nonempty and disjoint subsets $\mathcal{S}_1,\mathcal{S}_2\subset\mathcal{V}$, at least one of $\mathcal{S}_1$ or $\mathcal{S}_2$ is jointly r-reachable, where $r\in\mathbb{Z}_+$.

Remark 1: The concept of set reachability (or graph robustness) requires that there exist a sufficient number of agents in a node set (or a digraph) that possess enough in-neighbors from outside, either at each time step or within a certain time interval. These properties are important for achieving resilient consensus under adversarial attacks, since the MSR-based algorithms will eliminate a certain number of in-neighbors for all agents at each time step. It is also worth mentioning that the robust graph has a requirement on the minimum number of agents in the network. For an r-robust graph, we illustrate the cases of minimum agents required when $r=1,2,3,4$ in Fig. 1. When $r=1$, only two agents are needed to construct a 1-robust graph. However, when $r\geq 2$, it can be observed from Fig. 1 that $2r-1$ agents need to be involved to construct a robust graph. In summary, the minimum number of agents $N_{\min}(r)$ increases with the parameter r, and their quantitative relation is generalized as

$$N_{\min}(r)=\begin{cases}2, & \text{if } \ r=1,\\ 2r-1, & \text{otherwise.}\end{cases} \tag{1}$$

B. Dynamic Event-Triggering Consensus Strategy

Consider a heterogeneous MAS consisting of first-order and second-order agents and modelled by the time-invariant digraph $\mathcal{G}=(\mathcal{V},\mathcal{E})$. The sets of first-order and second-order agents are denoted as $\mathcal{N}_1$ and $\mathcal{N}_2$, respectively. The state update for first-order agents is presented as

$$x_i[k+1]=x_i[k]+\tau u_i[k], \tag{2}$$

where $x_i[k]$ and $u_i[k]$ represent the position and control input for first-order agent i at time step k, respectively, and τ is the sampling period.

The update rule for second-order agents is presented as

$$x_i[k+1]=x_i[k]+\tau v_i[k], \tag{3a}$$

$$v_i[k+1]=v_i[k]+\tau u_i[k], \tag{3b}$$

where $x_i[k]$, $v_i[k]$, and $v_i[k]$ represent the position, velocity and control input for second-order agent i at time step k, respectively.

Remark 2: Different from the studies [23]–[25] that use the zero-order hold method to discretize the system model, we use the forward difference approximation. This is because of the simplicity, causality, and low computational cost of the forward difference approximation method. The selection of the discretization method also aligns with the important factor considered in this paper: the consumption of computation and communication resources, which motivates us to design the dynamic event-triggering condition later. It should also be noted that no matter which model is chosen, the main results

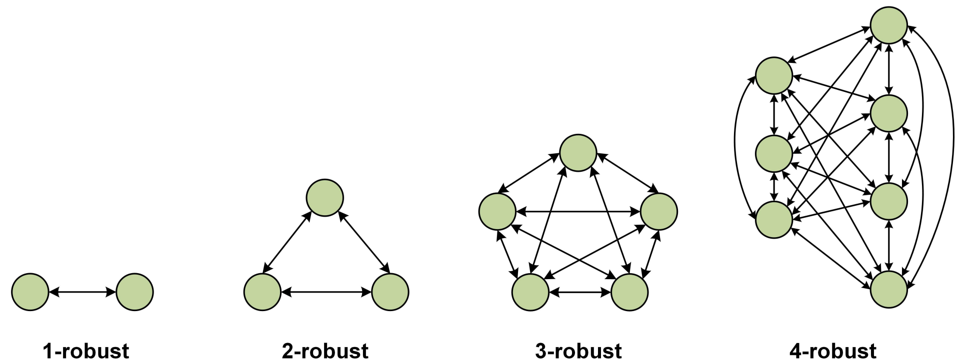

Fig. 1. The minimum number of agents required in an r-robust graph when $r = 1, 2, 3, 4$.

and numerical examples in this paper will not change essentially. In other words, both discretized models can achieve resilient consensus under their respective assumptions.

This study aims to make the positions of all healthy agents in the heterogeneous MAS asymptotically achieve consensus and the velocities of healthy second-order agents asymptotically approach zero. Based on this principle, the control strategy of first-order agents is devised according to their own positions and the auxiliary variable of position information from their in-neighbors. The control strategy of second-order agents is devised based on their own positions, velocities, and the auxiliary variable of position information from their in-neighbors. Specifically, the control strategy for healthy first-order agents is expressed as

$$u_i[k] = \sum_{j\in\mathcal{N}_i^+} a_{ij}[k]\left(\hat{x}_j[k] - x_i[k]\right), \quad i \in \mathcal{N}_1, \tag{4}$$

while the control strategy for healthy second-order agents is mathematically expressed as

$$u_i[k] = \sum_{j\in\mathcal{N}_i^+} a_{ij}[k]\left(\hat{x}_j[k] - x_i[k]\right) - \gamma v_i[k], \quad i \in \mathcal{N}_2, \tag{5}$$

where $a_{ij}[k]$ is the weight for the edge (j, i), $\hat{x}_j[k]$ is an auxiliary variable, which is defined as

$$\hat{x}_j[k] = x_j[t_m^j], \ k \in [t_m^j, t_{m+1}^j), \tag{6}$$

where $\{t_0^j, t_1^j, \ldots \in \mathbb{Z}_+\}$ are the transmission steps of agent j. In addition, the parameter γ refers to the control gain, which is a positive scalar.

Assumption 1: The parameter γ and the sampling period τ fulfill the following conditions:

$$\begin{gathered} 1)\ \tau \sum_{j\in\mathcal{N}_i^+} a_{ij}[k] \le 1; \quad 2)\ \tau^2 \sum_{j\in\mathcal{N}_i^+} a_{ij}[k] \le 1; \\ 3)\ \frac{1}{\tau} + \tau \sum_{j\in\mathcal{N}_i^+} a_{ij}[k] \le \gamma \le \frac{2}{\tau}. \end{gathered} \tag{7}$$

Remark 3: The main purpose of Assumption 1 is to ensure that the update of current states lies in the convex combination of previous states. Furthermore, it was revealed in [36] that this setting can effectively avoid the undesired oscillatory behavior.

To proceed, an internal dynamic variable $\delta_i[k]$ is incorporated to facilitate the dynamic event-triggering mechanism. Its state update follows

$$\delta_i[k+1] = (1-\theta_i)\delta_i[k] + \eta_i(\kappa\psi[k] - |e_i[k]|), \tag{8}$$

where $\psi[k]$ is a time-dependent threshold which satisfies $\psi[k] > 0$, $\forall k \in \mathbb{N}$ and $\lim_{k\to\infty}\psi[k] = 0$, the parameters κ, θ_i and η_i are positive scalars to be designed. Based on the internal dynamic variable $\delta_i[k]$, a novel dynamic event-triggering condition (DETC) is developed as

$$t_{m+1}^i = \min\left\{k > t_m^i : \xi_i\left(|e_i[k]| - \kappa\psi[k]\right) > \delta_i[k]\right\}, \tag{9}$$

where $e_i[k] = \hat{x}_i[k] - x_i[k+1]$ is a difference term, ξ_i is also a positive scalar to be designed.

Assumption 2: Regarding the parameters θ_i, η_i, and ξ_i in (8) and (9), the following three conditions hold.

$$1)\ \eta_i < \xi_i\theta_i; \quad 2)\ \eta_i + \theta_i < 1; \quad 3)\ \xi_i > \frac{1-\eta_i}{\theta_i}, \quad \forall i \in \mathcal{V}.$$

Remark 4: In the discrete-time domain, the minimum time interval between two triggered events defaults to one. Therefore, the Zeno phenomenon is automatically excluded for DETC (9). Moreover, Assumption 2 guarantees the positivity and convergence of $\delta_i[k]$, which play a crucial role in reaching resilient consensus for heterogeneous MAS. Compared to reference [26], our study balances convergence performance and trigger performance with the proposed DETC, i.e., achieving resilient consensus with a lower communication burden.

C. Attack Model and Resilient Consensus

In the context of malicious attack, agents in the heterogeneous MAS are classified into healthy agents (or healthy nodes) and faulty agents (or faulty nodes) according to the following definitions:

Definition 5 (Healthy agent [13]): An agent is said to be healthy if it sends its position $x_i[k]$ to all its out-neighbors at each time step k and uses the rules (2) or (3) for state update.

Definition 6 (Faulty agent [13]): An agent is said to be faulty if it sends its position $x_i[k]$ to all its out-neighbors at each time step k, but its state update is uncontrolled by the designed rule (manipulated by attackers).

We denote the sets of healthy and faulty agents as $\mathcal{H}$ and $\mathcal{F}$, respectively, where it holds $\mathcal{H} \subseteq \mathcal{V}$ and $\mathcal{F} := \mathcal{V} \backslash \mathcal{H}$. By synthesizing Definitions 5 and 6 with the heterogeneous MAS, the sets of healthy first-order and second-order agents are further denoted by $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively. It is worth mentioning that a healthy agent does not know the identity of faulty agent, while the attacker (faulty agent) may have the knowledge about the network topology to perpetrate a more targeted adversarial attack (e.g. attack vulnerable nodes). In addition, we consider a limitation on the maximum number of faulty agents in the in-neighbor set of each healthy agent and introduce the following attack model:

Definition 7 (f-local attack model [13]): The set of faulty agent $\mathcal{F}$ is said to satisfy the f-local attack model if there exist at most f faulty agents in the in-neighbor set of each healthy agent, i.e., $\left|\mathcal{N}_i^+[k] \cap \mathcal{F}\right| \leq f, \ \forall i \in \mathcal{V} \backslash \mathcal{F}, \ \forall k \in \mathbb{N}$, where $f \in \mathbb{N}$.

Subsequently, the resilient consensus problem to be tackled in this study is given below.

Definition 8 (Resilient consensus): The heterogeneous MAS is said to achieve resilient consensus if the following two conditions hold for all initial positions and velocities of agents and any possible faulty set:

1) *(Resilience)* All the healthy agents satisfy $x_i[k] \in \mathcal{S}, \ \forall k \in \mathbb{N}$, where $\mathcal{S}$ represents a safety interval and is mathematically expressed as

$$\begin{aligned} \mathcal{S} = \Big[& \min_{i \in \mathcal{H}} x_i[0] + \min_{i \in \mathcal{H}} \{0, (2-\tau\gamma)\tau v_i[0]\}, \\ & \max_{i \in \mathcal{H}} x_i[0] + \max_{i \in \mathcal{H}} \{0, (2-\tau\gamma)\tau v_i[0]\} \Big]. \end{aligned} \tag{10}$$

2) *(Consensus)* All the healthy agents satisfy $\lim_{k\to\infty} x_i[k] = C, \ \forall i \in \mathcal{H}$ and all the healthy second-order agents satisfy $\lim_{k\to\infty} v_l[k] = 0, \ \forall l \in \mathcal{H}_2$, where $C \in \mathbb{R}$.

Remark 5: The concept of "exact resilient consensus" is to distinguish the concept of "bounded resilient consensus" in event-based studies [25]–[27]. The former guarantees that the positions of healthy agents converge to an exact consensus value C, while the latter merely guarantees convergence to an error range. In addition, the resilience condition is to keep the positions of healthy agents always within a safe range. The consensus condition ensures that the positions of healthy agents asymptotically converge to the same state value in the presence of malicious attacks, while the velocities of healthy second-order agents asymptotically approach zero.

III. Main Results

This section presents a dynamic event-triggering resilient algorithm, gives the convex combination representation of the heterogeneous MAS, and provides sufficient graph conditions for ensuring resilient consensus.

A. HDE-MSR Algorithm

In this subsection, we design a novel *Heterogeneous Dynamic Event-triggering Mean-Subsequence-Reduced* (HDE-MSR) algorithm for addressing consensus problems in adversarial environments. The algorithm includes the six steps below.

1) *(Initializing parameters):* Initialize $\gamma > 0$, $\tau > 0$, $\xi_i > 0$, $\delta_i[0] > 0$, $\theta_i \in (0,1)$, and $\eta_i \in [0,1)$. These parameters satisfy the conditions given in Assumptions 1 and 2, with the in-neighbor set $\mathcal{N}_i^+$ in Assumption 1 being replaced by $\mathcal{R}_i^+[k]$.
2) *(Collecting in-neighbors' information):* The healthy agent i collects $x_i[k]$ and $\hat{x}_j[k]$ at each time step k, then sorts them in an ascending order.
3) *(Eliminating malicious states):* Compared with $x_i[k]$, agent i eliminates the f largest and smallest auxiliary variables $\hat{x}_j[k]$. If there are fewer than f auxiliary variables strictly larger or smaller than $x_i[k]$, then agent i will eliminate all these auxiliary variables. The removal of $\hat{x}_j[k]$ is achieved through $a_{ij}[k] = 0$.
4) *(Updating state value):* Denote $\mathcal{R}_i^+[k]$ as the set of retained auxiliary variables of agent i. If agent i is a healthy first-order agent, then it adopts

$$u_i[k] = \sum_{j \in \mathcal{R}_i^+[k]} a_{ij}[k] \left(\hat{x}_j[k] - x_i[k]\right) \tag{11}$$

for state update. If agent i is a healthy second-order agent, then it applies the following rule for state update:

$$u_i[k] = \sum_{j \in \mathcal{R}_i^+[k]} a_{ij}[k] \left(\hat{x}_j[k] - x_i[k]\right) - \gamma v_i[k]. \tag{12}$$

5) *(Updating auxiliary variable):* Agent i checks if DETC (9) is activated and sets $\hat{x}_i[k+1]$ as

$$\hat{x}_i[k+1] = \begin{cases} x_i[k+1], & \text{if DETC (9) activates,} \\ \hat{x}_i[k], & \text{otherwise.} \end{cases} \tag{13}$$

6) *(Updating dynamical variable):* Agent i updates its internal dynamical variable $\delta_i[k+1]$ according to (8).

Note that each healthy agent updates its position and velocity at each time step, while updates its auxiliary variable when DETC (9) triggers. The communication between agents also only occurs when the triggering condition is activated. In addition, an attractive feature of the proposed HDE-MSR algorithm is its anonymity, i.e., healthy agents do not need to identify any faulty node. Instead, they merely eliminate the possibly misleading information from their in-neighbors. Furthermore, different from the update schemes (4) and (5), the healthy agents only use the auxiliary variables remained after the HDE-MSR algorithm (denoted as $\mathcal{R}_i^+[k]$) for state update, as expressed in (11) and (12).

Assumption 3: Regarding the weight $a_{ij}[k]$ in (11) and (12) and for all time steps $k \in \mathbb{N}$, the following conditions hold.

1) $\sum_{j=1}^{n} a_{ij}[k] = 1$, where $n = |\mathcal{V}|$;
2) $a_{ij}[k] = 0$ if $j \notin \mathcal{R}_i^+[k]$;
3) $a_{ij}[k] \geq \omega, \ \forall j \in \mathcal{R}_i^+[k]$, where $\omega \in (0, 1/2]$ is a constant.

Remark 6: Since zero weights for some directed edges may lead to inability to achieve resilient consensus, it is necessary to set a lower bound w on $a_{ij}[k]$, though its specific value is not necessary to be determined.

B. Convex Combination Representation

In this subsection, we rewrite the dynamic equations (2) and (3) of the heterogeneous MAS as the following convex combination form. This operation is beneficial for convergence analysis.

Lemma 1: Under the control protocols (4) and (5), the position $x_i[k]$ for each healthy first-order agent $i \in \mathcal{H}_1$ can be represented as

$$x_i[k+1] = \left(1 - \tau \sum_{j \in \mathcal{R}_i^+[k]} a_{ij}[k]\right) x_i[k] + \tau \sum_{j \in \mathcal{R}_i^+[k]} a_{ij}[k]\hat{x}_j[k], \tag{14}$$

while the position $x_i[k]$ for each healthy second-order agent $i \in \mathcal{H}_2$ can be mathematically expressed as

$$x_i[k+1] = (2 - \gamma\tau)x_i[k] + \tau^2 \sum_{j \in \mathcal{R}_i^+[k]} a_{ij}[k-1]\hat{x}_j[k-1], + \left(\gamma\tau - 1 - \tau^2 \sum_{j \in \mathcal{R}_i^+[k]} a_{ij}[k-1]\right) x_i[k-1]. \tag{15}$$

Furthermore, the state $x_i[k+1]$ is the convex combination of $x_i[k]$ and $\hat{x}_j[k]$ in (14), while the position $x_i[k+1]$ is the convex combination of $x_i[k]$, $x_i[k-1]$, and $\hat{x}_j[k-1]$ in (15).

Proof: The proof of (14) is direct by synthesizing the control protocol (4) with the system (2), which is omitted here.

To derive (15), we first rewrite the system dynamics (3) as

$$x_i[k] = x_i[k-1] + \tau v_i[k-1], \tag{16a}$$

$$v_i[k] = v_i[k-1] + \tau u_i[k-1]. \tag{16b}$$

In addition, we give the control protocol (12) at time step $k-1$ as

$$u_i[k-1] = -\gamma v_i[k-1] - \beta[k-1], \tag{17}$$

with $\beta[k-1]$ being mathematically expressed as $\beta[k-1] = \sum_{j \in \mathcal{R}_i^+[k-1]} a_{ij}[k-1]\left(x_i[k-1] - \hat{x}_j[k-1]\right)$. Substituting (17) into (16b) yields

$$v_i[k] = (1 - \gamma\tau)v_i[k-1] - \tau\beta[k-1]. \tag{18}$$

Subsequently, we rewrite (16a) as

$$v_i[k-1] = \frac{x_i[k] - x_i[k-1]}{\tau}. \tag{19}$$

Substituting (18) and (19) into (3a) yields

$$x_i[k+1] = (2-\gamma\tau)x_i[k] + (\gamma\tau-1)x_i[k-1] + \tau^2\beta[k-1]. \tag{20}$$

Invoking the expression of $\beta[k-1]$, we eventually obtain (15). Moreover, the convex combination properties of (14) and (15) are guaranteed by Assumption 1. ■

C. Sufficient Graph Conditions

In this subsection, sufficient graph conditions are provided to ensure resilient consensus. The following lemma provides an important condition regarding sequence convergence.

Lemma 2 ([37]): Let $\{\varphi_k\}$ be a positive scalar sequence. Suppose that $\lim_{k\to\infty} \varphi_k = 0$. For $\rho \in (0, 1)$, it holds

$$\lim_{k\to\infty} \sum_{l=0}^{k} \rho^{k-l}\varphi_l = 0. \tag{21}$$

With Lemma 2, we further derive some vital properties of the dynamic variable $\delta_i[k]$ through the lemma below.

Lemma 3: Regarding the dynamic variable $\delta_i[k]$, the following two statements hold for all $i \in \mathcal{H}$:

$$1)\ \delta_i[k] > 0, \quad \forall k \in \mathbb{N}; \qquad 2)\ \lim_{k\to\infty} \delta_i[k] = 0.$$

Proof: We first prove the positivity of $\delta_i[k]$. Considering the implementation of DETC (9), one obtains

$$\xi_i\left(|e_i[k]| - \kappa\psi[k]\right) \le \delta_i[k], \tag{22}$$

which can be reorganized as

$$|e_i[k]| \le \frac{\delta_i[k]}{\xi_i} + \kappa\psi[k]. \tag{23}$$

Substituting (23) into (8) yields

$$\delta_i[k+1] \ge \left(1 - \theta_i - \frac{\eta_i}{\xi_i}\right)\delta_i[k]. \tag{24}$$

Note that (24) can be further bounded by

$$\delta_i[k] \ge \left(1 - \theta_i - \frac{\eta_i}{\xi_i}\right)^k \delta_i[0], \tag{25}$$

By invoking Assumption 2, one has $\delta_i[k] > 0$, which corresponds to the first condition in Lemma 3.

It remains to prove the convergence of $\delta_i[k]$. From (8), (22) and (25), one obtains

$$\delta_i[k+1] \le (1 - \theta_i + \frac{\eta_i}{\xi_i})\delta_i[k] + 2\kappa\psi[k]. \tag{26}$$

Since there are n agents in the network, Eq. (26) can be further bounded as

$$\delta_i[k+n] \le (1 - \theta_i + \frac{\eta_i}{\xi_i})^n\delta_i[k] + 2\kappa\sum_{\ell=0}^{n-1}(1 - \theta_i + \frac{\eta_i}{\xi_i})^{n-1-\ell}\psi[k+\ell]. \tag{27}$$

According to Lemma 2 and the first condition in Assumption 2, we eventually arrive at $\lim_{k\to\infty} \delta_i[k] = 0$, which corresponds to the second condition in Lemma 3. ■

Remark 7: When the parameter ξ_i goes to infinity, the DETC (9) will become the static event-triggering condition (SETC) presented in [25] and they will demonstrate the same triggering performance. In other words, the SETC can be regarded as a limiting case of the proposed DETC when ξ_i takes an excessively large value. For the parameter $\delta_i[k]$, a larger initial value $\delta_i[0]$ may result in larger inter-event time intervals. When $\delta_i[0]$ = 0, the DETC (9) will also become the SETC. Consequently, it can be concluded that the proposed

DETC generates fewer trigger steps compared to the existing SETC.

In what follows, let

$$\begin{aligned}\overline{M}[k] &= \max_{i\in\mathcal{H}}\left\{x_i[k-1], x_i[k]\right\},\\ \underline{m}[k] &= \min_{i\in\mathcal{H}}\left\{x_i[k-1], x_i[k]\right\}.\end{aligned} \tag{28}$$

The following lemma provides vital relations regarding these two quantities.

Lemma 4: Assume that the healthy agents adhere to the HDE-MSR algorithm for state update. Then, it holds

$$\overline{M}[k+1] \leq \overline{M}[k] + \tau\zeta_i[k], \tag{29a}$$
$$\underline{m}[k+1] \geq \underline{m}[k] - \tau\zeta_i[k], \quad \forall k\in\mathbb{N} \tag{29b}$$

for each healthy first-order agent $i \in \mathcal{H}_1$, where $\zeta_i[k] = \delta_i[k]/\xi_i + \kappa\psi[k]$. Furthermore, it holds

$$\overline{M}[k+1] \leq \overline{M}[k] + \tau^2\zeta_i[k-1], \tag{30a}$$
$$\underline{m}[k+1] \geq \underline{m}[k] - \tau^2\zeta_i[k-1], \quad \forall k\in\mathbb{Z}_+ \tag{30b}$$

for each healthy second-order agent $i \in \mathcal{H}_2$.

Proof: We start with the proof of (29a). Define $\epsilon_i[k] = \hat{x}_i[k] - x_i[k]$. From (13), one shows

$$\epsilon_i[k] = \begin{cases} 0, & \text{if DETC (9) activates,}\\ \hat{x}_i[k] - x_i[k+1], & \text{otherwise.}\end{cases} \tag{31}$$

Now from (22) and the definition of $e_i[k]$, one further obtains $|\epsilon_i[k]| \leq \zeta_i[k]$. Subsequently, the system dynamics (14) for healthy first-order agent $j \in \mathcal{H}_1$ can be upper bounded by

$$\begin{aligned} x_j[k+1] \leq & \left(1 - \tau\sum_{i\in\mathcal{R}_j^+[k]} a_{ji}[k]\right)\overline{M}[k]\\ & + \tau\sum_{i\in\mathcal{R}_j^+[k]} a_{ji}[k]\overline{M}[k] + \tau\sum_{i\in\mathcal{R}_j^+[k]} a_{ji}[k]\epsilon_i[k]\\ = & \overline{M}[k] + \tau\sum_{i\in\mathcal{R}_j^+[k]} a_{ji}[k]\epsilon_i[k]\\ \leq & \overline{M}[k] + \tau\zeta_i[k]. \end{aligned} \tag{32}$$

Note that (32) holds for any $j \in \mathcal{H}_1$. Therefore, one obtains $\overline{M}[k+1] \leq \overline{M}[k] + \tau\zeta_i[k]$. The proof of (29b) can be conducted by a similar procedure and thus omitted here.

Regarding the relation (30a), one combines (15) with the results of $\epsilon_i[k]$ and obtains

$$\begin{aligned} x_j[k+1] \leq & (2-\gamma\tau)\overline{M}[k] + (\gamma\tau - 1)\overline{M}[k]\\ & + \tau^2\sum_{i\in\mathcal{R}_j^+[k]} a_{ji}[k-1]\epsilon_i[k-1]\\ \leq & \overline{M}[k] + \tau^2\zeta_i[k-1]. \end{aligned} \tag{33}$$

Note that (33) holds for any $j \in \mathcal{H}_2$. Therefore, one has $\overline{M}[k+1] \leq \overline{M}[k] + \tau^2\zeta_i[k-1]$. Similar analysis can be conducted to derive (30b), which completes the proof. ∎

With Lemmas 1, 2, 3, and 4, it is time to give the convergence results for heterogeneous MASs in time-invariant and time-varying networks.

Theorem 1: Let Assumptions 1, 2, and 3 hold. When the underlying time-invariant network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is subject to the f-local attack model, resilient consensus with the HDE-MSR algorithm is achieved for heterogeneous MASs if $\mathcal{G}$ is $(2f+1)$-robust.

Proof: Here we only provide the convergence analysis for healthy second-order agents. For the proof of convergence for healthy first-order agents, please refer to the work [35].

We first show that the interval $\mathcal{S}$ satisfies the resilience condition. Synthesizing (3a), (3b) with (5) yields

$$\begin{aligned} x_i[k+1] = & a_{ii}[k-1]x_i[k-1] + (2-\tau\gamma)\tau v_i[k-1]\\ & + \tau^2\sum_{j\in\mathcal{R}_i^+[k-1]} a_{ij}[k-1]\hat{x}_j[k-1]. \end{aligned} \tag{34}$$

where $a_{ii}[k-1] = 1 - \tau^2\sum_{j\in\mathcal{R}_i^+[k-1]} a_{ij}[k-1]$. At time steps $k=0$ and $k=1$, it can be verified that

$$x_i[1] = x_i[0] + \tau v_i[0] \in \mathcal{S}, \tag{35}$$

and

$$\begin{aligned} x_i[2] = & a_{ii}[0]x_i[0] + (2-\tau\gamma)\tau v_i[0]\\ & + \tau^2\sum_{j\in\mathcal{R}_i^+[0]} a_{ij}[0]\hat{x}_j[0] \in \mathcal{S}. \end{aligned} \tag{36}$$

Note that the initial velocities of the faulty agents do not appear in Eq. (35) or Eq. (36). On the other hand, the initial positions of the faulty agents can have influence on the healthy agents. At time step k, we suppose that $x_i[k] \in \mathcal{S}, \hat{x}_i[k] \in \mathcal{S}$ holds for each healthy second-order agent $i \in \mathcal{H}_2$. Subsequently, for agent i, its in-neighbors in $\mathcal{R}_i^+[k]$ take values only in $\mathcal{S}$, since there are at most f faulty agents with state values outside $\mathcal{S}$, and these values will be eliminated through the HDE-MSR algorithm. Thus from Eq. (13) and Eq. (34), we obtain $x_i[k+1] \in \mathcal{S}$ and $\hat{x}_i[k+1] \in \mathcal{S}$, which indicates that $\mathcal{S}$ is the safety interval for healthy second-order agents.

We next prove the consensus condition. Consider a healthy second-order agent $i \in \mathcal{H}_2$. For any positive scalar μ and for $k \in [0, k_l]$, let

$$\begin{aligned} \mathcal{A}_1(k, k_l, \mu) &= \left\{j\in\mathcal{V} : x_j[k_l] > \overline{M}[k] - \mu\right\},\\ \mathcal{A}_2(k, k_l, \mu) &= \left\{j\in\mathcal{V} : x_j[k_l] < \underline{m}[k] + \mu\right\}. \end{aligned} \tag{37}$$

Furthermore, define $\mathcal{B}_1(k, k_l, \mu) = \mathcal{A}_1(k, k_l, \mu) \cap \mathcal{H}$ and $\mathcal{B}_2(k, k_l, \mu) = \mathcal{A}_2(k, k_l, \mu) \cap \mathcal{H}$, which merely capture the healthy agents in $\mathcal{A}_1(k, k_l, \mu)$ and $\mathcal{A}_2(k, k_l, \mu)$, respectively.

Let $D[k] = \overline{M}[k] - \underline{m}[k]$. Then, the consensus condition in Definition 8 can be rewritten as $\lim_{k\to\infty} D[k] = 0$ and $\lim_{k\to\infty} v_i[k] = 0, \quad \forall i \in \mathcal{H}_2$. We first prove the former part of the consensus condition. Suppose $D[k] > 0$ at some time step k. Let $\mu_0 = D[k]/2$. Then, the two sets $\mathcal{B}_1(k, k, \mu_0)$ and $\mathcal{B}_2(k, k, \mu_0)$ are disjoint due to $\overline{M}[k] - \mu_0 = \underline{m}[k] + \mu_0$. Furthermore, both of them are nonempty since $\mathcal{B}_1(k, k, \mu_0)$ and $\mathcal{B}_2(k, k, \mu_0)$ possess some healthy agents whose positions are $\overline{M}[k]$ or $\underline{m}[k]$. As the underlying network $\mathcal{G}$ is $(2f+1)$-robust, there exists a healthy agent $i \in \mathcal{H}$ in either $\mathcal{B}_1(k, k, \mu_0)$ or $\mathcal{B}_2(k, k, \mu_0)$ that possesses at least $2f+1$ in-neighbors outside of its respective set.

Suppose $i \in \mathcal{B}_1(k, k, \mu_0)$. Since the underlying network is subject to the f-local attack model, at most f in-neighbors of agent i are from the faulty set $\mathcal{F}$ and the remaining $f+1$ in-neighbors are healthy agents. Considering the implementation of proposed resilient strategy, at least one of these $f+1$ in-neighbors has its position upper bounded by $\overline{M}[k]-\mu_0$, and its position will be remained for the update of agent i. Synthesizing this result with (30a), the position $x_i[k+1]$ follows

$$\begin{aligned} x_i[k+1] &\leq (1-\omega)\left(\overline{M}[k]+\tau^2\zeta_i[k-1]\right) \\ &\quad + \omega(\overline{M}[k]-\mu_0+\tau^2\zeta_i[k-1]) \\ &= \overline{M}[k]-\omega\mu_0+\tau^2\zeta_i[k-1], \end{aligned} \tag{38}$$

where the inequality is established by placing the weight $1-\omega$ on $\overline{M}[k]+\tau^2\zeta_i[k-1]$. Notice that the relation (38) also applies to healthy agents in $\mathcal{V}\backslash\mathcal{A}_1(k,k,\mu_0)$, since they will utilize their own positions for state update.

If the healthy agent $i \in \mathcal{B}_2(k,k,\mu_0)$ possesses at least $2f+1$ in-neighbors outside of its set, one obtains

$$\begin{aligned} x_i[k+1] &\geq (1-\omega)\left(\underline{m}[k]-\tau^2\zeta_i[k-1]\right) \\ &\quad + \omega(\underline{m}[k]+\mu_0-\tau^2\zeta_i[k-1]) \\ &= \underline{m}[k]+\omega\mu_0-\tau^2\zeta_i[k-1], \end{aligned} \tag{39}$$

which also applies to healthy agents in $\mathcal{V}\backslash\mathcal{A}_2(k,k,\mu_0)$.

Next, one defines $\mu_1=\omega\mu_0-\tau^2\zeta_i[k-1]$. Based on (38) and (39), at least one healthy agent in $\mathcal{B}_1(k,k,\mu_0)$ has its position less than or equal to $\overline{M}[k]-\mu_1$ at time step $k+1$, or at least one healthy agent in $\mathcal{B}_2(k,k,\mu_0)$ has its position greater than or equal to $\underline{m}[k]+\mu_1$ at time step $k+1$, or both. This fact indicates

$$\begin{aligned} &|\mathcal{B}_1(k,k+1,\mu_1)|+|\mathcal{B}_2(k,k+1,\mu_1)| \\ &<|\mathcal{B}_1(k,k,\mu_0)|+|\mathcal{B}_2(k,k,\mu_0)|. \end{aligned} \tag{40}$$

Notice that $\mu_1<\mu_0$. Thus, $\mathcal{B}_1(k,k+1,\mu_1)$ and $\mathcal{B}_2(k,k+1,\mu_1)$ are also disjoint. Once they are nonempty, one concludes that there exists a healthy agent $j\in\mathcal{H}$ in either $\mathcal{B}_1(k,k+1,\mu_1)$ or $\mathcal{B}_2(k,k+1,\mu_1)$ that has at least $2f+1$ in-neighbors outside its respective set. Similar to the aforementioned analysis, one assumes $j\in\mathcal{B}_1(k,k+1,\mu_1)$. Then, the position $x_j[k+2]$ follows

$$\begin{aligned} x_j[k+2] &\leq (1-\omega)(\overline{M}[k+1]+\tau^2\zeta_i[k]) \\ &\quad + \omega(\overline{M}[k]-\mu_1+\tau^2\zeta_i[k]) \\ &\leq (1-\omega)(\overline{M}[k]+\tau^2\zeta_i[k]+\tau^2\zeta_i[k-1]) \\ &\quad + \omega(\overline{M}[k]-\omega\mu_0+\tau^2\zeta_i[k]+\tau^2\zeta_i[k-1]) \\ &= \overline{M}[k]-\omega^2\mu_0+\tau^2(\zeta_i[k]+\zeta_i[k-1]). \end{aligned} \tag{41}$$

Otherwise, if $j\in\mathcal{B}_2(k,k+1,\mu_1)$, then the position $x_j[k+2]$ is lower bounded by

$$\begin{aligned} x_j[k+2] &\geq (1-\omega)(\underline{m}[k+1]-\tau^2\zeta_i[k]) \\ &\quad + \omega(\underline{m}[k]+\mu_1-\tau^2\zeta_i[k]) \\ &\geq (1-\omega)(\underline{m}[k]-\tau^2\zeta_i[k]-\tau^2\zeta_i[k-1]) \\ &\quad + \omega(\underline{m}[k]+\omega\mu_0-\tau^2\zeta_i[k]-\tau^2\zeta_i[k-1]) \\ &= \underline{m}[k]+\omega^2\mu_0-\tau^2(\zeta_i[k]+\zeta_i[k-1]). \end{aligned} \tag{42}$$

Consequently, it yields

$$\begin{aligned} &|\mathcal{B}_1(k,k+2,\mu_2)|+|\mathcal{B}_2(k,k+2,\mu_2)| \\ &<|\mathcal{B}_1(k,k+1,\mu_1)|+|\mathcal{B}_2(k,k+1,\mu_1)|, \end{aligned} \tag{43}$$

where $\mu_2=\omega^2\mu_0-\tau^2(\zeta_i[k]+\zeta_i[k-1])$.

By recursion, one defines $\mu_{k'}=\omega^{k'}\mu_0-\tau^2\sum_{\ell=-1}^{k'-2}\zeta_i[k+\ell],\ \forall k'\in\mathbb{Z}_+$. If $\mathcal{B}_1(k,k+k',\mu_{k'})$ and $\mathcal{B}_2(k,k+k',\mu_{k'})$ are nonempty, one can repeat the aforementioned analysis and show that at least one of $\mathcal{B}_1(k,k+k',\mu_{k'})$ and $\mathcal{B}_2(k,k+k',\mu_{k'})$ will shrink at the next time step. Due to $|\mathcal{B}_1(k,k,\mu_0)|+|\mathcal{B}_2(k,k,\mu_0)|\leq n$, at least one of $\mathcal{B}_1(k,k+n,\mu_n)$ and $\mathcal{B}_2(k,k+n,\mu_n)$ is empty. Suppose $\mathcal{B}_1(k,k+n,\mu_n)=\varnothing$. From (37), one obtains $\overline{M}[k+n]\leq\overline{M}[k]-\mu_n$. By invoking Lemma 4, one obtains $\underline{m}[k+n]\geq\underline{m}[k]-\sum_{\ell=-1}^{n-2}\zeta_i[k+\ell]$. Therefore,

$$\begin{aligned} D[k+n] &\leq D[k]-\mu_n+\sum_{\ell=-1}^{n-2}\zeta_i[k+\ell] \\ &= (1-\frac{\omega^n}{2})D[k]+2\sum_{\ell=-1}^{n-2}\zeta_i[k+\ell]. \end{aligned} \tag{44}$$

For any $\sigma\in\mathbb{N}$, one shows

$$\begin{aligned} D[k+\sigma n] &\leq (1-\frac{\omega^n}{2})^\sigma D[k] \\ &\quad +2\sum_{\ell=-1}^{n-2}\sum_{\lambda=0}^{\sigma-1}(1-\frac{\omega^n}{2})^{\sigma-1-\lambda}\zeta_i[k+\ell+\lambda n] \\ &= (1-\frac{\omega^n}{2})^\sigma D[k] \\ &\quad +2\sum_{\ell=-1}^{n-2}\sum_{\lambda=0}^{\sigma-1}(1-\frac{\omega^n}{2})^{\sigma-1-\lambda}\frac{\delta_i[k+\ell+\lambda n]}{\xi_i} \\ &\quad +2\kappa\sum_{\ell=-1}^{n-2}\sum_{\lambda=0}^{\sigma-1}(1-\frac{\omega^n}{2})^{\sigma-1-\lambda}\psi[k+\ell+\lambda n]. \end{aligned} \tag{45}$$

Note that the right hand of (45) consists of three terms. Now one analyzes the convergence of each term individually. First, it is clear that

$$\lim_{\sigma\to\infty}(1-\frac{\omega^n}{2})^\sigma D[k]=0 \tag{46}$$

due to $\omega\in(0,1)$ and $D[k]>0$. Subsequently, by invoking Lemmas 2 and 3, one derives

$$\lim_{\sigma\to\infty}\sum_{\ell=-1}^{n-2}\sum_{\lambda=0}^{\sigma-1}(1-\frac{\omega^n}{2})^{\sigma-1-\lambda}\psi[k+\ell+\lambda n]=0 \tag{47}$$

and

$$\lim_{\sigma\to\infty}\sum_{\ell=-1}^{n-2}\sum_{\lambda=0}^{\sigma-1}(1-\frac{\omega^n}{2})^{\sigma-1-\lambda}\frac{\delta_i[k+\ell+\lambda n]}{\xi_i}=0. \tag{48}$$

Synthesizing the above results yields $\lim_{\sigma\to\infty}D[k+\sigma n]=0$. Thus, one eventually arrives at $\lim_{k\to\infty}D[k]=0$. This fact indicates that resilient consensus is ensured.

Finally, one reveals that the velocities of all healthy second-order agents asymptotically approach zero, i.e., $\lim_{k\to\infty}v_i[k]=0,\ \forall i\in\mathcal{H}_2$. When the positions of healthy second-order agents achieve consensus, one obtains $\lim_{k\to\infty}(x_i[k+1]-x_i[k])=0,\quad i\in\mathcal{H}_2$. Taking limits on both sides of (3a) yields

$$\lim_{k\to\infty}(x_i[k+1]-x_i[k])=\tau\lim_{k\to\infty}v_i[k]. \tag{49}$$

Therefore, we eventually arrive at $\lim_{k\to\infty} v_i[k] = 0,\ \forall i \in \mathcal{H}_2$, which completes the proof. ■

Theorem 2: Let Assumptions 1, 2, and 3 hold. When the underlying time-varying network $\mathcal{G}[k] = (\mathcal{V}, \mathcal{E}[k])$ is subject to the f-local attack model, resilient consensus with the HDE-MSR algorithm is achieved for heterogeneous MASs if $\mathcal{G}[k]$ is jointly $(2f+1)$-robust.

Proof: The proof of the resilience condition is the same as Theorem 1 and thus omitted here. For the consensus condition, similar to the proof of Theorem 1, one constructs two nonempty and disjoint sets $\mathcal{B}_1(k, k_h, \mu_{k_h-k})$ and $\mathcal{B}_2(k, k_h, \mu_{k_h-k})$. As the underlying network $\mathcal{G}[k]$ is jointly $(2f+1)$ robust, there exists a time step k_j in each time interval $[k_h, k_{h+1})$ such that there exists a healthy agent $i \in \mathcal{H}$ in either $\mathcal{B}_1(k, k_h, \mu_{k_h-k})$ or $\mathcal{B}_2(k, k_h, \mu_{k_h-k})$ that has at least $2f+1$ in-neighbors outside its respective set. Subsequently, one derives

$$\begin{aligned} &\left|\mathcal{B}_1(k, k_{h+1}, \mu_{k_{h+1}-k})\right| + \left|\mathcal{B}_2(k, k_{h+1}, \mu_{k_{h+1}-k})\right| \\ &< \left|\mathcal{B}_1(k, k_h, \mu_{k_h-k})\right| + \left|\mathcal{B}_2(k, k_h, \mu_{k_h-k})\right| \end{aligned} \tag{50}$$

if both $\mathcal{B}_1(k, k_h, \mu_{k_h-k})$ and $\mathcal{B}_2(k, k_h, \mu_{k_h-k})$ are nonempty. Therefore, at least one of $\mathcal{B}_1(k, k+Tn, \mu_{Tn})$ and $\mathcal{B}_2(k, k+Tn, \mu_{Tn})$ is empty due to $|\mathcal{B}_1(k,k,\mu_0)| + |\mathcal{B}_2(k,k,\mu_0)| \le n$, where T is the maximum length of time intervals $[k_h, k_{h+1})$. For both of these two cases, one shows

$$\begin{aligned} D[k+nT] &\le D[k] - \mu_{nT} + \sum_{\ell=-1}^{nT-2} \zeta_i[k+\ell] \\ &= \left(1 - \frac{\omega^{nT}}{2}\right) D[k] + 2\sum_{\ell=-1}^{nT-2} \zeta_i[k+\ell], \end{aligned} \tag{51}$$

which results in $\lim_{\sigma\to\infty} D[k+\sigma nT] = 0$. Thus, we eventually reach $\lim_{k\to\infty} D[k] = 0$.

It remains to prove the velocity convergence of healthy second-order agents. By adopting a similar analysis to (49), we eventually arrive at $\lim_{k\to\infty} v_i[k] = 0,\ \forall i \in \mathcal{H}_2$. This completes the proof of resilient consensus in time-varying networks. ■

IV. Case Studies

In this section, three case studies are given to validate the theoretical findings. In the first case, we achieve resilient consensus for healthy agents in time-invariant networks. With the introduction of dynamic event-triggering mechanism, we find that the consumption of communication resources is significantly mitigated. In the second case, resilient consensus is further guaranteed over time-varying networks, which merely requires network topology to be jointly robust. A comparative case study is conducted in the third case to show the dominance of the proposed DETC-based resilient consensus scheme compared with the existing methods based on the static event-triggering condition (SETC).

A. Dynamic Event-Triggering Resilient Consensus in Time-Invariant Networks

Consider a heterogeneous MAS consisting of seven agents and described by Fig. 2, where $\mathcal{H}_1 = \{1,2,3,4\}$ and $\mathcal{H}_2 = \{5,6,7\}$. Let Agent 1 be a faulty first-order agent, whose motion is expressed as $x_1[k] = 5 \times \sin(k/5) + 5$. The initial states of seven agents are denoted as $[x_1[0], \cdots, x_7[0]]^{\mathrm{T}} = [7,7,6,8,-5,8,6]^{\mathrm{T}}$ and $[v_1[0], \cdots, v_7[0]]^{\mathrm{T}} = [0,0,0,0,3,-2,-1]^{\mathrm{T}}$, respectively. Regarding the dynamic event-triggering mechanism, we let $\delta_i[0] = 20$, $\kappa = 0.01$ and $\psi[k] = \mathrm{e}^{-\varepsilon k}$ with $\varepsilon = 0.01$. In view of Assumption 1, we let $\tau = 0.2$ and $\gamma = 6$. To satisfy Assumption 2, we select $\xi_i = 10, \eta_i = 0.6, \theta_i = 0.35,\ \forall i \in \mathcal{V}$. Furthermore, the reason we set $\delta_i[0] = 20$ and $\xi_i = 10$ is that these parameters originate from the design of DETC, and their settings align with the references [28], [30], both of which are based on the dynamic event-triggering mechanism.

Figs. 3(a) and 3(b) display the positions and velocities

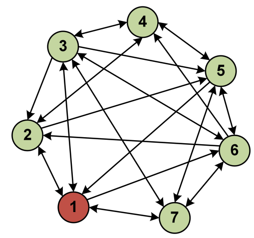

Fig. 2. A 3-robust digraph which fulfills the 1-local attack model.

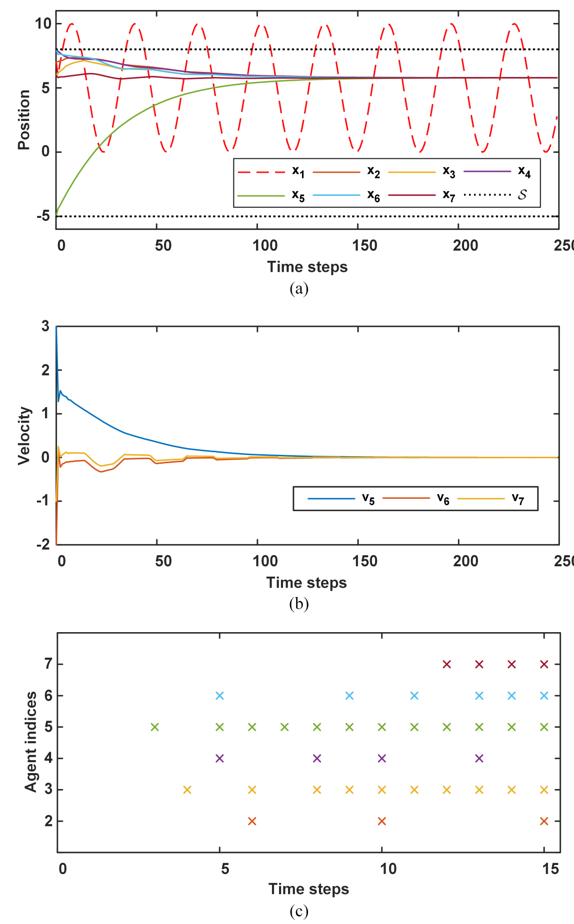

Fig. 3. Convergence results and triggering behaviors using the proposed HDE-MSR algorithm in time-invariant networks: (a) trajectories of all agents; (b) velocities of second-order agents; (c) event instants with DETC (9).

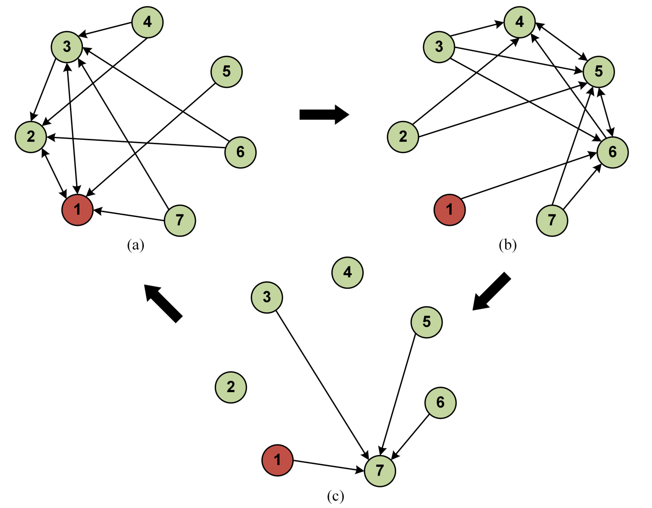

Fig. 4. A jointly 3-robust digraph which fulfills the 1-local attack model: (a) $\mathcal{G}_1$; (b) $\mathcal{G}_2$; (c) $\mathcal{G}_3$.

of agents with the proposed HDE-MSR algorithm in time-invariant networks. It can be seen that though Agent 1 has been manipulated by attackers and performs a sinusoidal motion, the positions of all healthy agents achieve resilient consensus within the safety interval $\mathcal{S}$. Meanwhile, the velocities of healthy second-order agents asymptotically converges to zero. These results validate Theorem 1. Furthermore, Fig. 3(c) reveals that the communication between agents only occurs occasionally within the first fifteen time steps, which indicates that the heavy communication burden is mitigated.

B. Dynamic Event-Triggering Resilient Consensus in Time-Varying Networks

In this subsection, resilient consensus is further guaranteed in time-varying networks to relax the graph robustness requirement at each time step. To this end, we consider a jointly 3-robust digraph $\mathcal{G}[k]$ with seven agents, where $\mathcal{H}_1 = \{1, 2, 3, 4\}$ and $\mathcal{H}_2 = \{5, 6, 7\}$. The network topology is shown in Fig. 4 and switches as follows:

$$\mathcal{G}[k] = \begin{cases} \mathcal{G}_1, & k = 3m, \\ \mathcal{G}_2, & k = 3m+1, \quad\quad m \in \mathbb{N} \\ \mathcal{G}_3, & k = 3m+2. \end{cases} \tag{52}$$

Other simulation settings are the same as that in Section IV-A.

Figs. 5(a) and 5(b) illustrate the positions and velocities of agents with the proposed HDE-MSR algorithm in time-varying networks. Although faulty agent 1 deviates from the normal state update, the positions of healthy agents reach resilient consensus within the safety interval $\mathcal{S}$, and the velocities of healthy second-order agents asymptotically approach zero. These results validate Theorem 2. Moreover, Fig. 5(c) reveals that the communication between agents also occurs occasionally in this case, indicating that the heavy communication burden is mitigated. Furthermore, it is worth mentioning that though none of the three digraphs in Fig. 4 is a robust graph, resilient consensus is still ensured when the system suffers from the 1-local attack model. This fact means that the conventional requirement on network topology is relaxed by adopting the jointly 3-robust digraph.

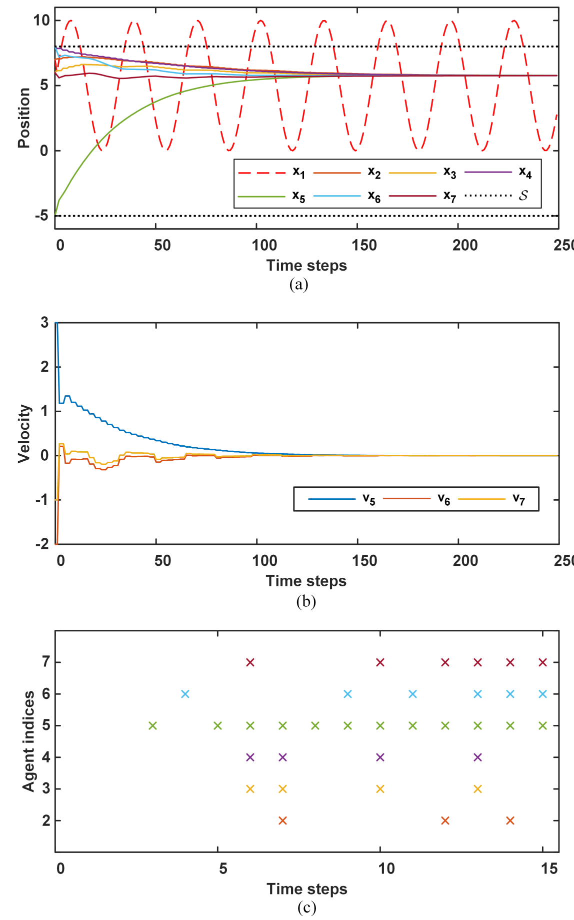

Fig. 5. Convergence results and triggering behaviors using the proposed HDE-MSR algorithm in time-varying networks: (a) trajectories of all agents; (b) velocities of second-order agents; (c) event instants with DETC (9).

C. Comparison between SETC and DETC

In [25], a static event-triggering condition (SETC) was introduced as

$$t_{m+1}^i = \min\left\{k > t_m^i : |e_i[k]| > c_0 + c_1 \mathrm{e}^{-\alpha k}\right\}, \tag{53}$$

where c_0, c_1, and α are positive scalars to be designed. On this basis, the E-MSR algorithm was adopted to guarantee bounded resilient consensus for heterogeneous MASs. In this subsection, a comparative analysis is conducted to show the advantages of DETC in contrast to the existing SETC. For SETC (53), we select $c_0 = 0$, $c_1 = 0.01$, $\alpha = 0.01$. The reason we set $c_0 = 0$ is that we expect to achieve exact resilient consensus in both SETC and DETC cases. For DETC (9), the settings are identical as that in Section IV-A. Moreover, by setting these parameters, we also keep the exponential term in both SETC and DETC as $0.01 \times \mathrm{e}^{-0.01k}$.

Fig. 6 shows the convergence results and triggering behaviors using the E-MSR algorithm [25] in time-invariant network. By setting $c_0 = 0$, we can still ensure resilient consensus for all healthy agents. However, their triggering

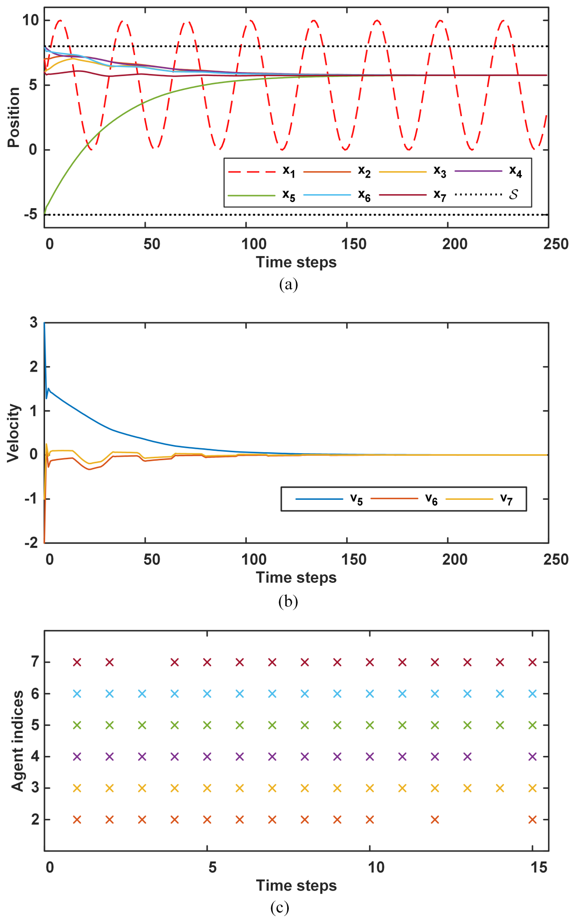

Fig. 6. Convergence results and triggering behaviors using the E-MSR algorithm [25] in time-invariant networks: (a) trajectories of all agents; (b) velocities of second-order agents; (c) event instants with SETC (53).

conditions are frequently violated during the initial fifteen time steps. In contrast, Fig. 3(c) and Fig. 5(c) exhibit that the triggering functions for healthy agents are occasionally activated with DETC (9). Under different triggering mechanisms and network topologies, the triggering times for all healthy agents within the first 250 time steps are recorded in TABLE I. By comparing the first and second rows of TABLE I, it is observed that the triggering times for all healthy agents decrease to varying degrees with DETC (9). By comparing the second and third rows of TABLE I, we find that the triggering times for healthy agents have further decreased in time-varying networks. This is due to the sparse connection between agents. This result suggests that a comprehensive consideration of dynamic event-triggering mechanism and time-varying network is beneficial for a significant reduction in the number of triggered events. In addition, Fig. 3(a) indicates that the convergence rate of DETC (9) is similar to that of Fig. 6(a). The numerical results suggest that the proposed DETC-based resilient control scheme is more advantageous in mitigating the heavy communication burden compared to current SETC-based resilient algorithms, while keeping a similar convergence rate.

TABLE I
COMPARISON OF THE EVENT TRIGGERING BEHAVIOR BETWEEN SETC (53) AND DETC (9) IN TIME-INVARIANT AND TIME-VARYING NETWORK TOPOLOGIES (WITHIN 250 TIME STEPS).

Result	Network topology	Event-triggering mechanism	The number of triggered events					
			ag.2	ag.3	ag.4	ag.5	ag.6	ag.7
Fig. 6	Time-invariant	SETC (53)	150	127	149	178	113	102
Fig. 3	Time-invariant	DETC (9)	140	122	138	174	106	90
Fig. 5	Time-varying	DETC (9)	78	56	78	160	84	115

V. CONCLUSIONS

Via a novel resilient control strategy, this paper solves the resilient consensus problem for heterogeneous MASs when the network is subject to the f-local attack model. The proposed HDE-MSR algorithm is implemented to reach resilient consensus in time-invariant and switching network topologies with reduced communication overheads. In time-invariant networks, it is proven that $(2f+1)$-robustness is a sufficient condition on the network topology to ensure resilient consensus, while in time-varying networks, the sufficient condition is the joint $(2f+1)$-robustness. Finally, three case studies are presented to validate the effectiveness and dominance of the HDE-MSR algorithm.

The investigation of resilient consensus for more complex situations and systems will be our future research directions, e.g., multi-dimensional space, leader-follower MASs, and more general modelling of heterogeneous MASs. Application-oriented resilient coordination tasks, such as resilient containment and resilient distributed optimization, will also be considered.

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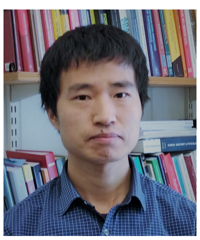

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