Dynamic Event-Triggering Resilient Consensus for Heterogeneous MASs Against Malicious Attacks

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Abstract: Based on a novel dynamic event-triggering mechanism, this work studies the resilient consensus problem for heterogeneous multi-agent systems (MASs) consisting of first-order and second-order agents. An internal dynamic variable is introduced to adjust the triggering threshold flexibly and facilitate the construction of the dynamic event-triggering condition (DETC). Subsequently, a heterogeneous dynamic event-triggering mean-subsequence-reduced (HDE-MSR) algorithm is developed, which ensures that the positions of all healthy agents achieve consensus on the same value and the velocities of all healthy second-order agents asymptotically approach zero. Finally, an illustrative example is provided to validate the theoretical findings.

Key Words: Heterogeneous MAS, resilient consensus, dynamic event-triggering mechanism, malicious attack

1 Introduction

The increasing concern for cyber security in recent years has been accompanied by the attention paid to solving the consensus problem for multi-agent systems (MASs) against malicious attacks. Open communications via shared networks are vulnerable to potential attacks, thereby causing irreparable losses. The heterogeneity of agents increases the difficulty of designing resilient algorithms. Another concern is the heavy communication burden, which is caused by frequent interactions between agents. Motivated by the aforementioned observations, this work investigates the resilient consensus problem for heterogeneous MASs and devises a dynamic event-triggering controller to guarantee exact resilient consensus with reduced communication overheads.

2 Problem Formulation and Algorithm Design

2.1 System Model

Consider a heterogeneous MAS consisting of first-order and second-order agents and modelled by the time-invariant digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The sets of first-order and second-order agents are denoted as $\mathcal{N}_1$ and $\mathcal{N}_2$, respectively. The state update for first-order agents is presented as

$$x_i[k+1] = x_i[k] + \tau u_i[k], \tag{1}$$

where $x_i[k]$ and $u_i[k]$ represent the position and control input for first-order agent i at time step k, respectively, and τ is the sampling period.

The update rule for second-order agents is presented as

$$x_i[k+1] = x_i[k] + \tau v_i[k], \tag{2a}$$
$$v_i[k+1] = v_i[k] + \tau u_i[k], \tag{2b}$$

where $x_i[k]$, $v_i[k]$, and $v_i[k]$ represent the position, velocity and control input for second-order agent i at time step k, respectively.

2.2 Formulation of the Resilient Consensus Problem

The resilient consensus problem to be tackled in this work is formulated as below.

Definition 1 *The heterogeneous MAS is said to achieve exact resilient consensus if the following two conditions hold for all initial positions and velocities of nodes and any possible faulty set:*

1) *(Safety) All the healthy agents satisfy $x_i[k] \in \mathcal{S}$, $\forall k \in \mathbb{N}$, where $\mathcal{S}$ represents a safety interval.*
2) *(Exact consensus) All the healthy agents satisfy $\lim_{k\to\infty} |x_i[k] - x_j[k]| = 0$, $\forall i, j \in \mathcal{H}$ and all healthy second-order agents satisfy $\lim_{k\to\infty} v_l[k] = 0$, $\forall l \in \mathcal{H}_2$.*

2.3 Design of Dynamic Event-Triggering Strategy

The control strategy for first-order agents is designed as

$$u_i[k] = -\sum_{j\in\mathcal{N}_i^+} a_{ij}[k]\left(x_i[k] - \hat{x}_j[k]\right), \;\; i \in \mathcal{H}_1, \tag{3}$$

while the control strategy for healthy second-order agents is mathematically expressed as

$$u_i[k] = -\gamma v_i[k] - \sum_{j\in\mathcal{N}_i^+} a_{ij}[k]\left(x_i[k] - \hat{x}_j[k]\right), \;\; i \in \mathcal{H}_2, \tag{4}$$

where $a_{ij}[k]$ is the weight of edge (j, i), $\hat{x}_j[k]$ is an auxiliary variable, which is defined as

$$\hat{x}_j[k] = x_j[t_m^j], \; k \in [t_m^j, t_{m+1}^j), \tag{5}$$

where $\{t_0^j, t_1^j, \ldots \in \mathbb{Z}_+\}$ is the transmission times of agent j. In addition, the parameter γ refers to the control gain, which is a positive scalar.

Motivated by [1], an internal dynamic variable $\delta_i[k]$ is incorporated to facilitate a dynamic event-triggering mechanism. Its state update follows

$$\delta_i[k+1] = (1-\theta_i)\delta_i[k] + \eta_i(\psi[k] - |e_i[k]|), \tag{6}$$

where $\psi[k]$ is a time-dependent threshold which satisfies $\psi[k] > 0$, $\forall k \in \mathbb{N}$ and $\lim_{k\to\infty} \psi[k] = 0$, the parameters θ_i and η_i are positive scalars. Based on the internal dynamic variable $\delta_i[k]$, a novel dynamic event-triggering condition (DETC) is developed as

$$t^i_{m+1} = \min\left\{k > t^i_m : \xi_i\left(|e_i[k]| - \psi[k]\right) > \delta_i[k]\right\}, \quad (7)$$

where ξ_i is also a positive scalar.

2.4 HDE-MSR

To eliminate the potential threats for each healthy agent, a novel HDE-MSR algorithm is developed in this work, which includes the following five steps.

1) *(Collecting in-neighbors' information):* The healthy agent i collects $x_i[k]$ and $\hat{x}_j[k]$ at each time step k, then sorts them in an ascending order.
2) *(Eliminating malicious states):* Compared with $x_i[k]$, agent i eliminates the f largest and smallest auxiliary variables $\hat{x}_j[k]$ in the sorted list. If there are less than f of the values strictly larger or smaller than $x_i[k]$, then all of them will be eliminated. The removal of $\hat{x}_j[k]$ is achieved through $a_{ij}[k] = 0$.
3) *(Updating state value):* Denote $\mathcal{R}^+_i[k]$ as the set of retained auxiliary variables of agent i. If Agent i is a healthy first-order agent, then it adopts

$$u_i[k] = -\sum_{j\in\mathcal{R}^+_i[k]} a_{ij}[k]\left(x_i[k] - \hat{x}_j[k]\right) \quad (8)$$

for state update. If Agent i is a healthy second-order agent, then it applies the following rule for state update:

$$u_i[k] = -\gamma v_i[k] - \sum_{j\in\mathcal{R}^+_i[k]} a_{ij}[k]\left(x_i[k] - \hat{x}_j[k]\right). \quad (9)$$

4) *(Updating auxiliary variable):* Agent i checks if DETC (7) is activated and sets $\hat{x}_i[k+1]$ as

$$\hat{x}_i[k+1] = \begin{cases} x_i[k+1], & \text{if DETC (7) activates,} \\ \hat{x}_i[k], & \text{otherwise.} \end{cases} \quad (10)$$

5) *(Updating dynamical variable):* Agent i updates its interval dynamical variable $\delta_i[k+1]$ according to (6).

3 Illustrative Example

Consider a heterogeneous MAS consisting of seven agents and described by Fig. 1, where $\mathcal{H}_1 = \{1,2,3,4\}$ and $\mathcal{H}_2 = \{5,6,7\}$. Let Agent 1 be a faulty first-order agent, whose motion is expressed as $x_1[k] = 5 \times \sin(k/5) + 5$. The initial positions and velocities of seven agents are denoted as $[x_1[0], \cdots, x_7[0]]^{\mathrm{T}} = [7,7,6,8,-5,8,6]^{\mathrm{T}}$ and $[v_1[0], \cdots, v_7[0]]^{\mathrm{T}} = [0,0,0,0,3,-2,-1]^{\mathrm{T}}$, respectively. For the dynamic event-triggering mechanism, we let $\psi[k] = \mathrm{e}^{-\varepsilon k}$ with $\varepsilon = 0.01$ and let $\delta_i[0] = 15$ and select $\xi_i = 10, \eta_i = 0.6, \theta_i = 0.35$, $\forall i \in \mathcal{V}$. In addition, we choose the sampling period as $\tau = 0.2$ and the control gain as $\gamma = 6$.

Figs. 2(a) and 2(b) display the positions and velocities of agents with the proposed HDE-MSR algorithm in time-invariant networks. It can be observed that despite the influence of Agent 1, the positions of healthy agents achieve exact

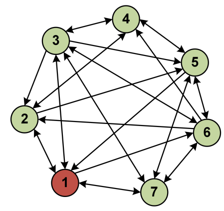

Fig. 1: A 3-robust digraph.

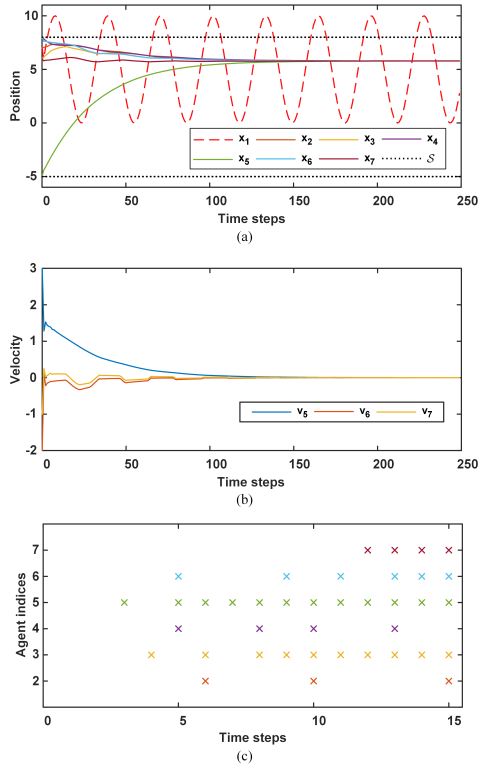

Fig. 2: Convergence results and triggering behaviors using the proposed HDE-MSR algorithm in time-invariant networks: (a) trajectories of all agents; (b) velocities of second-order agents; (c) event instants with DETC (7).

resilient consensus within the safety interval $\mathcal{S}$. Meanwhile, the velocities of healthy second-order agents asymptotically converges to zero. Furthermore, the result of event instants with DETC (7) is shown in Fig. 2(c), from which we observe that the information interaction between agents only occurs occasionally within the first fifteen time steps. This result indicates that the heavy communication burden is lightened.

4 Conclusions

This work solves the resilient consensus problem for heterogeneous MASs when the network is subject to malicious attack. The proposed HDE-MSR algorithm is implemented to reach resilient consensus with reduced communication overheads. Future work includes extending the existing results to higher-order or nonlinear MASs.

References

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