Resilient Consensus Through Dynamic Event-Triggered Mechanism

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***Abstract*—In this brief, the resilient consensus problem for multi-agent systems (MASs) is addressed based on the dynamic event-triggered (DE) mechanism, when the network is subject to malicious attacks. A dynamic variable is introduced in the DE mechanism to adjust the threshold dynamically. Via the proposed dynamic event-triggered mean-subsequence-reduced (DE-MSR) algorithm, all cooperative agents are guaranteed to reach an agreement on the same consensus value, so that the MAS achieves resilient consensus despite the influence of some noncooperative agents in the network. Compared to existing event-based resilient algorithms, the proposed DE-MSR algorithm is superior in reducing communication overheads while maintaining a resilient consensus. Finally, a comparative case study is conducted to validate the theoretical findings.**

***Index Terms*—Multi-agent system, resilient consensus, dynamic variable, event-triggered mechanism, malicious attack.**

I. INTRODUCTION

THE INCREASING concern for cyber security in recent years has been accompanied by the attention paid to the multi-agent consensus problem against malicious attacks [1], [2], [3]. Open communications via shared networks are vulnerable to potential attacks, which may cause irreparable losses. Thus, an essential problem in multi-agent systems (MASs) is resilient consensus [4], which involves cooperative agents aiming to reach an agreement on their states, while some noncooperative agents attempt to destroy this goal. These adversaries are outside the scope of preset interactive rules and may lead to the overall system crash [5], [6], [7], [8].

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Specific to resilient consensus problems, cooperative agents usually seek for agreement through local interaction. The impact of attacks can be mitigated through various techniques. Among them, a family of algorithms called *Mean-Subsequence-Reduced* (MSR) has attracted tremendous attention due to its fault-tolerant and light-weight properties. In [9], a weighted MSR (W-MSR) algorithm was adopted to achieve resilient consensus against malicious attacks, with each cooperative agent filtering the suspicious values received from its in-neighbors. Variants of this algorithm include the double-integrator position-based MSR (DP-MSR) [10], sliding-window MSR (SW-MSR) [11], and quantized-weighted MSR (QW-MSR) [12] algorithms. Furthermore, the MSR-type algorithms have been widely applied in hybrid MASs [13], continuous-time MASs [14], and impulsive control [15].

In addition to malicious attacks, another concern for resilient consensus is the heavy communication burden. Specifically, most existing MSR-type algorithms require each agent to interact with its in-neighbors at each time step to obtain their state values for update. To reduce communication overheads, an event-based MSR (E-MSR) algorithm was proposed in [16], which involves an event-triggered mechanism to restrict the interaction between agents. Subsequently, this brief was extended to a multi-dimensional space [17], and combined with self-triggered [18] and quantized [19] mechanisms. Nevertheless, all these promising results merely seek for approximate resilient consensus, which guarantees that the MAS converges to a given error range instead of an exact consensus value. Additionally, agents may be constrained by predefined event conditions, which are often fixed and cannot adapt to changing environmental conditions or agent behaviors.

Motivated by the above observations, this brief aims to design a dynamic event-triggered (DE) controller for the MAS to achieve resilient consensus, such that all cooperative agents converge to the same consensus value. The proposed resilient scheme is able to adjust the triggering condition dynamically, which shares high adaptability and flexibility. The innovations of this brief are highlighted as follows:

1) In contrast to DE studies [20], [21], [22] in attack-free scenarios, this brief incorporates the DE mechanism to solve the resilient consensus problem of MASs over directed graphs, when the network is subject to malicious attacks.
2) The frequent communication between neighbors, as required in [9], [10], [11], is relaxed via a novel

dynamic event-triggered MSR (DE-MSR) algorithm, which saves system resources and reduces communication overheads.

3) Compared to existing event-based resilient algorithms [16], [17], [18], [19], the proposed DE-MSR algorithm guarantees that all cooperative agents converge to the same consensus value rather than an error range, so that the MAS achieves *exact* resilient consensus.

The rest of this brief is organized as follows. Section II introduces some preliminaries, together with formulating the resilient consensus problem. Section III presents the main results for achieving resilient consensus with the DE mechanism. We validate the main results through a comparative case study in Section IV. Finally, Section V concludes this brief.

II. Preliminaries and Problem Formulation

A. Graph Theory

Consider a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The set of nodes is given by $\mathcal{V}$. The set of edges is denoted by $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The edge $(j, i) \in \mathcal{E}$ indicates that node i can receive information from node j. Since $\mathcal{G}$ is a digraph, we define the sets of in-neighbors and out-neighbors as $\mathcal{V}_i^{\text{in}} = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$ and $\mathcal{V}_i^{\text{out}} = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$, respectively.

Some essential notions with respect to (w.r.t.) sets and graphs are presented to characterize the resilience for the MAS.

Definition 1 [9]: Consider a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. A nonempty subset $\mathcal{S} \subseteq \mathcal{V}$ is r-reachable if there exists an agent $i \in \mathcal{S}$ such that $|\mathcal{V}_i^{\text{in}} \setminus \mathcal{S}| \geq r$, where $r \in \mathbb{Z}_{>0}$.

Definition 2 [9]: Consider a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. $\mathcal{G}$ is r-robust if for each pair of nonempty and disjoint subsets $\mathcal{S}_1, \mathcal{S}_2 \subseteq \mathcal{V}$, at least one of them is r-reachable, where $r \in \mathbb{Z}_{>0}$.

B. System Model

Consider a single-integrator MAS described by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. We denote $x_i[t] \in \mathbb{R}$ as the state for agent i at time step $t \in \mathbb{Z}_{\geq 0}$. The state update for each agent $i \in \mathcal{V}$ is based on a prescribed rule, which is mathematically expressed as

$$x_i[t+1] = x_i[t] + u_i[t], \tag{1}$$

where $u_i[t]$ represents the control input.

In this brief, we consider the situation that there exist some *noncooperative agents* in the network. They will disseminate malicious information to all of their out-neighbors. Such misbehaviors may destroy the operation of other *cooperative agents*, and disrupt system security. The definitions of cooperative and noncooperative agents are presented as follows.

Definition 3 [9]: An agent is said to be cooperative if it utilizes the prescribed rule to update its state. Let the set of cooperative agents be $\mathcal{C} \subseteq \mathcal{V}$. An agent is noncooperative if it does not adhere to the prescribed update rule, but utilizes some other protocols for state update at some time steps. Let the set of noncooperative agents be $\mathcal{A} := \mathcal{V} \setminus \mathcal{C}$.

The attack model is further defined below to quantify the distribution of malicious attacks.

Definition 4 [9]: For $f \in \mathbb{Z}_{\geq 0}$, the noncooperative set $\mathcal{A}$ is said to be an f-local model if $|\mathcal{V}_i^{\text{in}} \cap \mathcal{A}| \leq f, \ \forall i \in \mathcal{V} \setminus \mathcal{A}$.

C. Problem Formulation

This brief aims to tackle the following problem, despite the influence of noncooperative agents.

Problem 1: Consider a MAS described by digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Suppose that at most f noncooperative agents exist in the in-neighbor set of each agent in the network. For any initial states of agents, determine graph conditions and design controllers such that the conditions below are satisfied:

- *Resilience condition:* For each cooperative agent $i \in \mathcal{C}$, it holds $x_i[t] \in \mathcal{S}$, $\forall t \in \mathbb{Z}_{\geq 0}$, where $\mathcal{S} = [\min_{i \in \mathcal{C}} x_i[0], \max_{i \in \mathcal{C}} x_i[0]]$.
- *Consensus condition:* For each pair of cooperative agents $i, j \in \mathcal{C}$, it holds $\lim_{t \to \infty} |x_i[t] - x_j[t]| = 0$.

Remark 1: The aforementioned two conditions can be interpreted as: (1) the state values of cooperative agents should always remain in a safety interval $\mathcal{S}$ despite the influence of noncooperative agents, and (2) the ultimate states of any two cooperative agents should be totally identical.

D. Dynamic Event-Triggered Protocol

Compared to existing event-based resilient algorithms [16], [17], [18], [19], the dynamic triggering pattern is superior in reducing communication overheads. In this subsection, we develop a DE consensus scheme. To start with, we introduce the event-triggered protocol [16] as below:

$$u_i[t] = -\sum_{j \in \mathcal{V}_i^{\text{in}}} a_{ij}[t]\big(x_i[t] - \hat{x}_j[t]\big), \tag{2}$$

where $\hat{x}_j[t] \in \mathbb{R}$ denotes the last broadcast state value of agent j, which is defined as $\hat{x}_j[t] = x_j[t_m^j]$, $t \in [t_m^j, t_{m+1}^j)$, with $\{t_0^j, t_1^j, \ldots, \in \mathbb{Z}_{>0}\}$ being the sequence of triggering times. Following the works [16], [17], [18], a static event-triggered function (SETF) is designed as

$$t_{m+1}^j = \min\big\{t > t_m^j : \big|e_j[t]\big| > \epsilon_0 + \epsilon_1 \mathrm{e}^{-\alpha t}\big\}, \tag{3}$$

where $e_j[t] = x_j[t+1] - \hat{x}_j[t]$ is the relative error between the updated state $x_j[t+1]$ and the auxiliary variable $\hat{x}_j[t]$, and ϵ_0, ϵ_1, and α are positive scalars.

For the dynamic triggering case, a dynamic variable $\xi_i[t] \in \mathbb{R}$ is introduced and its state update adheres to

$$\xi_i[t+1] = (1 - \psi_i)\xi_i[t] + \phi_i(\vartheta[t] - |e_i[t]|), \tag{4}$$

where ψ_i and ϕ_i are parameters to be designed, $\vartheta[t]$ is a positive threshold which converges exponentially to zero. With $\xi_i[t]$, agent i determines its triggering times $\{t_m^i\}_{m=1}^{\infty}$ by the following dynamic event-triggered function (DETF):

$$t_{m+1}^i = \min\big\{t > t_m^i : \zeta_i(|e_i[t]| - \vartheta[t]) > \xi_i[t]\big\}, \tag{5}$$

where ζ_i is also a parameter to be designed.

Assumption 1: For any $i \in \mathcal{V}$, the parameters ψ_i, ϕ_i, and ζ_i satisfy the following conditions:

$$1)\ \zeta_i > \frac{1 - \phi_i}{\psi_i}; \quad 2)\ \phi_i + \psi_i < 1; \quad 3)\ \phi_i < \zeta_i \psi_i. \tag{6}$$

These conditions play a crucial role in the proof of Lemma 2. Based on the designed DETF, we further develop a *Dynamic Event-triggered Mean-Subsequence-Reduced* (DE-MSR) algorithm. Taking the state update of cooperative agent

Algorithm 1 DE-MSR Algorithm

Require: Parameter $f \in \mathbb{Z}_{\geq 0}$
Initialize $x_i[0]$, $\hat{x}_i[0]$, $\xi_i[0] > 0$, ψ_i, ζ_i, and ϕ_i;
for $t = 0, 1, \ldots,$ the cooperative agent $i \in \mathcal{C}$ **do**
  Receive the initial or updated $\hat{x}_j[t]$ from $j \in \mathcal{V}_i^{\text{in}}[t]$, and sort them in ascending order;
  if less than f of the values in the sorted list are strictly larger [resp. smaller] than $x_i[t]$ **then**
    Eliminate all these values that are strictly larger [resp. smaller] than $x_i[t]$;
  else
    Eliminate the f largest [resp. smallest] values;
  end if
  Denote $\mathcal{R}_i^{\text{in}}[t]$ as the set of retained in-neighbors;
  Update the state according to

$$x_i[t+1] = x_i[t] - \sum_{j \in \mathcal{R}_i^{\text{in}}[t]} a_{ij}[t]\big(x_i[t] - \hat{x}_j[t]\big); \tag{7}$$

  if DETF (5) triggers **then**
    Update $\hat{x}_i[t+1]$ as $\hat{x}_i[t+1] = x_i[t+1]$;
    Send $\hat{x}_i[t+1]$ to the agents in $\mathcal{V}_i^{\text{out}}[t]$;
  else
    Set $\hat{x}_i[t+1]$ as $\hat{x}_i[t+1] = \hat{x}_i[t]$.
  end if
end for

$i \in \mathcal{C}$ as an example, the main steps of the DE-MSR algorithm are shown in Algorithm 1. Different from the update scheme (2), the cooperative agents will only utilize the state values of in-neighbors retained after Algorithm 1 for update, as expressed in (7).

Assumption 2: The weight $a_{ij}[t]$ in (7) satisfies the following conditions:

1) $\sum_{j=1}^{n} a_{ij}[t] = 1, \ \forall t \in \mathbb{Z}_{\geq 0}$;
2) $a_{ij}[t] = 0$ whenever $j \notin \mathcal{V}_i^{\text{in}}[t], \ t \in \mathbb{Z}_{\geq 0}$;
3) there exists a positive scalar $w > 0$ such that $a_{ij}[t] \geq w$, $\forall j \in \mathcal{V}_i^{\text{in}}[t], \ t \in \mathbb{Z}_{\geq 0}$.

III. Main Results

In this section, we investigate the efficiency of the DE-MSR algorithm and the consensus of the system (1) equipped with DETF (5), when the noncooperative set is subject to the f-local model.

Lemma 1 [23]: Let $\gamma \in (0, 1)$ and let $\{\varphi_k\}$ be a sequence consisting of positive scalar values. Suppose that $\lim_{k\to\infty} \varphi_k = 0$. Then

$$\lim_{k\to\infty} \sum_{l=0}^{k} \gamma^{k-l} \varphi_l = 0. \tag{8}$$

The following lemma provides essential properties for the dynamic variable $\xi_i[t]$.

Lemma 2: Let Assumption 1 holds. Suppose that the cooperative agents adopt the DE-MSR algorithm for update. For any $i \in \mathcal{C}$, the dynamic variable $\xi_i[t]$ satisfies the following statements for all $t \in \mathbb{Z}_{\geq 0}$:

$$1) \quad \xi_i[t] > 0; \qquad 2) \quad \lim_{t\to\infty} \xi_i[t] = 0.$$

Proof: We start the proof with the first statement. Since DETF (5) is adopted, we have

$$\zeta_i(|e_i[t]| - \vartheta[t]) \leq \xi_i[t]. \tag{9}$$

Synthesizing (9) with (4), we obtain

$$\xi_i[t+1] \geq \left(1 - \psi_i - \frac{\phi_i}{\zeta_i}\right)\xi_i[t]. \tag{10}$$

By recursion, it is derived that the dynamic variable $\xi_i[t]$ is always strictly greater than zero:

$$\xi_i[t] \geq \left(1 - \psi_i - \frac{\phi_i}{\zeta_i}\right)^t \xi_i[0] > 0, \tag{11}$$

where the last inequality holds due to (6).

For the second statement, we first rewrite (9) as

$$|e_i[t]| \leq \frac{\xi_i[t]}{\zeta_i} + \vartheta[t]. \tag{12}$$

Combining the results of (4), (11), and (12), we obtain

$$\xi_i[t+1] \leq \left(1 - \psi_i + \frac{\phi_i}{\zeta_i}\right)\xi_i[t] + 2\vartheta[t]. \tag{13}$$

By recursion, we further derive

$$\begin{aligned}\xi_i[t+n] \leq &\left(1 - \psi_i + \frac{\phi_i}{\zeta_i}\right)^n \xi_i[t] \\ &+ 2\sum_{l=0}^{n-1}\left(1 - \psi_i + \frac{\phi_i}{\zeta_i}\right)^{n-1-l} \vartheta[t+l]. \end{aligned} \tag{14}$$

In view of the third condition in (6) and Lemma 1, we have $\lim_{t\to\infty} \xi_i[t] = 0$, which completes the proof. ■

To proceed, denote $M[t] = \max_{i\in\mathcal{C}} x_i[t]$ and $m[t] = \min_{i\in\mathcal{C}} x_i[t]$ as the maximum and minimum state for cooperative agents at time step t. Let $L[t] = M[t] - m[t] > 0$ be the difference between these two quantities. In order to establish the consensus condition, we need to prove that $L[t]$ asymptotically converges to zero.

In order to find a unified form to express the auxiliary variable $\hat{x}_j[t]$, we define $\sigma_j[t] = \hat{x}_j[t] - x_j[t]$. By invoking the DE-MSR algorithm, we further obtain

$$\sigma_j[t] = \begin{cases} 0, & \text{if DETF (5) triggers,} \\ \hat{x}_j[t] - x_j[t+1], & \text{otherwise.} \end{cases} \tag{15}$$

By invoking (9), we know

$$\left|\sigma_j[t]\right| \leq \frac{\xi_j[t]}{\zeta_j} + \vartheta[t]. \tag{16}$$

Lemma 3: Suppose that each normal agent follows DE-MSR algorithm. For any $t \in \mathbb{Z}_{\geq 0}$ and for any $i \in \mathcal{C}$, the following relations hold:

$$\begin{aligned} M[t+1] &\leq M[t] + \frac{\xi_i[t]}{\zeta_i} + \vartheta[t], \\ m[t+1] &\geq m[t] - \frac{\xi_i[t]}{\zeta_i} - \vartheta[t]. \end{aligned} \tag{17}$$

Based on the results provided by Lemmas 1, 2, and 3, we are now ready to state the main result for the DETF case.

Theorem 1: Consider an MAS consisting of n agents and described by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Let Assumptions 1 and 2 hold. Suppose that $\mathcal{G}$ is $(2f+1)$-robust and the noncooperative set $\mathcal{A}$ satisfies the f-local model. Then, exact resilient consensus is achieved if the cooperative agents adopt the proposed DE-MSR algorithm for state update.

Proof: For any $\varepsilon \in \mathbb{R}$ and $0 \le t \le t'$, define two node sets as $\mathcal{P}_M(t,t',\varepsilon) = \{j \in \mathcal{V} : x_j[t'] > M[t] - \varepsilon\}$ and $\mathcal{P}_m(t,t',\varepsilon) = \{j \in \mathcal{V} : x_j[t'] < m[t] + \varepsilon\}$. Furthermore, denote $\mathcal{Q}_M(t,t',\varepsilon) = \mathcal{P}_M(t,t',\varepsilon) \cap \mathcal{C}$ and $\mathcal{Q}_m(t,t',\varepsilon) = \mathcal{P}_m(t,t',\varepsilon) \cap \mathcal{C}$ as two node sets that contain the cooperative agents in $\mathcal{P}_M(t,t',\varepsilon)$ and $\mathcal{P}_m(t,t',\varepsilon)$, respectively.

As mentioned above, we need to prove that $L[t]$ asymptotically converges to zero. By contradiction, we assume $L[t] > 0$ at some time step t. Let $\varepsilon_0 = L[t]/2$ and establish two nonempty and disjoint node sets $\mathcal{Q}_M(t,t,\varepsilon_0)$ and $\mathcal{Q}_m(t,t,\varepsilon_0)$. The reasons for nonempty and disjoint properties are that there exist some cooperative agents with state values being $M[t]$ or $m[t]$ in both sets, and $M[t] - \varepsilon_0 > m[t] + \varepsilon_0$.

Since $\mathcal{G}$ is $(2f+1)$-robust and the noncooperative set $\mathcal{A}$ satisfies the f-local model, either $\mathcal{Q}_M(t,t,\varepsilon_0)$ or $\mathcal{Q}_m(t,t,\varepsilon_0)$ contains a cooperative agent i that has at least $2f+1$ in-neighbors outside of its respective set. Without loss of generality, we assume $i \in \mathcal{Q}_M(t,t,\varepsilon_0)$. Through the DE-MSR algorithm, at least one of these $2f+1$ in-neighbors will be retained for the state update of agent i. Thus, the upper bound of $x_i[t+1]$ satisfies

$$\begin{aligned} x_i[t+1] &\le (1-w)\left(M[t] + \frac{\xi_i[t]}{\zeta_i} + \vartheta[t]\right) \\ &\quad + w\left(M[t] - \varepsilon_0 + \frac{\xi_i[t]}{\zeta_i} + \vartheta[t]\right) \\ &= M[t] - w\varepsilon_0 + \frac{\xi_i[t]}{\zeta_i} + \vartheta[t]. \end{aligned} \tag{18}$$

It should be noted that the inequality (18) also holds for the cooperative agents in $\mathcal{V}\backslash\mathcal{P}_M(t,t,\varepsilon_0)$. If $i \in \mathcal{Q}_m(t,t,\varepsilon_0)$, we have $x_i[t+1] \ge m[t] + w\varepsilon_0 - (\xi_i[t]/\zeta_i + \vartheta[t])$, which also holds for the cooperative agents in $\mathcal{V}\backslash\mathcal{P}_m(t,t,\varepsilon_0)$.

Define $\varepsilon_1 = w\varepsilon_0 - (\xi_i[t]/\zeta_i + \vartheta[t])$. Based on (18), we know that at least one cooperative node exists in $\mathcal{Q}_M(t,t,\varepsilon_0)$ with its state value less than or equal to $M[t] - \varepsilon_1$ at time step $t+1$, or at least one cooperative node exists in $\mathcal{Q}_m(t,t,\varepsilon_0)$ with its state value greater than or equal to $m[t] + \varepsilon_1$ at time step $t+1$. Thus, at least one of $\mathcal{Q}_M(t,t+1,\varepsilon_1) \subsetneq \mathcal{Q}_M(t,t,\varepsilon_0)$ and $\mathcal{Q}_m(t,t+1,\varepsilon_1) \subsetneq \mathcal{Q}_m(t,t,\varepsilon_0)$ holds.

Notice that $\mathcal{Q}_M(t,t+1,\varepsilon_1)$ and $\mathcal{Q}_m(t,t+1,\varepsilon_1)$ are disjoint due to $\varepsilon_1 < \varepsilon_0$. If they are nonempty, we can conduct a similar analysis as above and conclude that either $\mathcal{Q}_M(t,t+1,\varepsilon_1)$ or $\mathcal{Q}_m(t,t+1,\varepsilon_1)$ contains a cooperative agent j, whose state value is upper bounded by

$$\begin{aligned} x_j[t+2] &\le (1-w)\left(M[t+1] + \frac{\xi_i[t+1]}{\zeta_i} + \vartheta[t+1]\right) \\ &\quad + w\left(M[t] - \varepsilon_1 + \frac{\xi_i[t+1]}{\zeta_i} + \vartheta[t+1]\right) \\ &\le M[t] - w^2\varepsilon_0 + \sum_{\lambda=0}^{1}\left(\frac{\xi_i[t+\lambda]}{\zeta_i} + \vartheta[t+\lambda]\right), \end{aligned} \tag{19}$$

if $j \in \mathcal{Q}_M(t,t+1,\varepsilon_1)$. Likewise, we have $x_j[t+2] \ge m[t] + w^2\varepsilon_0 - \sum_{\lambda=0}^{1}(\frac{\xi_i[t+\lambda]}{\zeta_i} + \vartheta[t+\lambda])$ if $j \in \mathcal{Q}_m(t,t+1,\varepsilon_1)$. Define $\varepsilon_2 = w^2\varepsilon_0 - \sum_{\lambda=0}^{1}(\frac{\xi_i[t+\lambda]}{\zeta_i} + \vartheta[t+\lambda])$. Therefore, at least one of $\mathcal{Q}_M(t,t+2,\varepsilon_2) \subsetneq \mathcal{Q}_M(t,t+1,\varepsilon_1)$ and $\mathcal{Q}_m(t,t+2,\varepsilon_2) \subsetneq \mathcal{Q}_m(t,t+1,\varepsilon_1)$ holds.

By recursion, we can define $\varepsilon_k = w^k\varepsilon_0 - \sum_{\lambda=0}^{k-1}(\frac{\xi_i[t+\lambda]}{\zeta_i} + \vartheta[t+\lambda])$, $\forall k \in \mathbb{Z}_{>0}$ and conclude that all cooperative agents will move out from $\mathcal{Q}_M(t,t+n,\varepsilon_n)$ or $\mathcal{Q}_m(t,t+n,\varepsilon_n)$ eventually due to $|\mathcal{Q}_M(t,t,\varepsilon_0)| + |\mathcal{Q}_m(t,t,\varepsilon_0)| \le n$. Thus, at least one of $\mathcal{Q}_M(t,t+n,\varepsilon_n)$ and $\mathcal{Q}_m(t,t+n,\varepsilon_n)$ is empty. Without loss of generality, we assume $\mathcal{Q}_M(t,t+n,\varepsilon_n) = \varnothing$. Then, we have $M[t+n] \le M[t] - \varepsilon_n$. In view of Lemma 3, we obtain $m[t+n] \ge m[t] - \sum_{\lambda=0}^{n-1}(\frac{\xi_i[t+\lambda]}{\zeta_i} + \vartheta[t+\lambda])$. Combining these two results yields

$$\begin{aligned} L[t+n] &\le L[t] - \varepsilon_n + \sum_{\lambda=0}^{n-1}\left(\frac{\xi_i[t+\lambda]}{\zeta_i} + \vartheta[t+\lambda]\right) \\ &= \left(1 - \frac{w^n}{2}\right)L[t] + 2\sum_{\lambda=0}^{n-1}\left(\frac{\xi_i[t+\lambda]}{\zeta_i} + \vartheta[t+\lambda]\right). \end{aligned} \tag{20}$$

For any $\sigma \in \mathbb{N}$, we have

$$\begin{aligned} L[t+\sigma n] &\le \left(1 - \frac{w^n}{2}\right)^{\sigma} L[t] \\ &\quad + 2\sum_{\lambda=0}^{n-1}\sum_{l=0}^{\sigma-1}\left(1 - \frac{w^n}{2}\right)^{\sigma-1-l}\frac{\xi_i[t+\lambda+ln]}{\zeta_i} \\ &\quad + 2\sum_{\lambda=0}^{n-1}\sum_{l=0}^{\sigma-1}\left(1 - \frac{w^n}{2}\right)^{\sigma-1-l}\vartheta[t+\lambda+ln]. \end{aligned} \tag{21}$$

According to Lemmas 1 and 2, we know

$$\lim_{\sigma\to\infty}\sum_{\lambda=0}^{n-1}\sum_{l=0}^{\sigma-1}\left(1 - \frac{w^n}{2}\right)^{\sigma-1-l}\vartheta[t+\lambda+ln] = 0, \tag{22}$$

and

$$\lim_{\sigma\to\infty}\sum_{\lambda=0}^{n-1}\sum_{l=0}^{\sigma-1}\left(1 - \frac{w^n}{2}\right)^{\sigma-1-l}\frac{\xi_i[t+\lambda+ln]}{\zeta_i} = 0. \tag{23}$$

Thus, we eventually obtain $\lim_{t\to\infty} L[t] = 0$, which means that exact resilient consensus is guaranteed for the MAS. ■

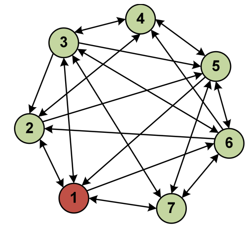
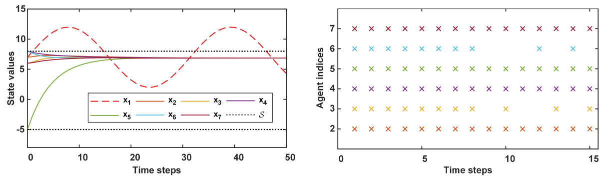


Fig. 1. A 3-robust digraph with seven agents.

IV. Case Study

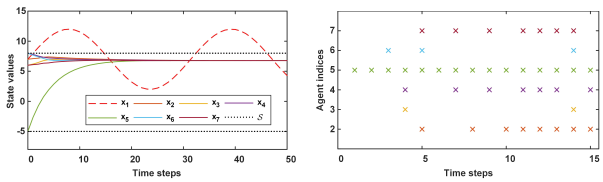
In this section, a comparative case study is provided to validate the theoretical results.

Consider the network of an MAS consisting of seven agents illustrated in Fig. 1. Notice that the network is a 3-robust digraph. According to Theorem 1, it can tolerate at most one noncooperative agent in the in-neighbors of each agent. Thus, we postulate that the network satisfies the 1-local model and let Agent 1 be noncooperative, whose motion is described by $x_1[t] = 5\times\sin(t/5) + 7$. The initial state values of seven agents are denoted as $\mathrm{x}[0] = [x_1[0], \ldots, x_7[0]]^{\mathrm{T}} = [7, 7, 6, 8, -5, 8, 6]^{\mathrm{T}}$. For the SETF case, the related parameters are chosen as $\epsilon_0 = 5\times10^{-4}$, $\epsilon_1 = 0.01$, $\alpha = 0.01$. For the DETF case, we let $\vartheta[t] = \mathrm{e}^{-\beta t}$ with $\beta = 0.01$, and set



(a) (b)

Fig. 2. Trajectories and triggering behaviors of agents using SETF (3).



(a) (b)

Fig. 3. Trajectories and triggering behaviors of agents using DETF (5).

TABLE I
COMPARISON OF TRIGGERING TIMES BETWEEN SETF (3) AND DETF (5)

Event-triggered mechanism	Event counts (within 250 time steps)					
	ag.2	ag.3	ag.4	ag.5	ag.6	ag.7
SETF (3)	33	24	29	26	18	22
DETF (5)	26	15	20	26	10	14

$\xi_i[0] = 20$. To satisfy Assumption 1, the parameters ψ_i, ϕ_i, and ζ_i are chosen as $\psi_i = 0.35$, $\phi_i = 0.6$, $\zeta_i = 10$. Note that the selection of the above parameters holds for all $i \in \mathcal{V}$.

Firstly, Figs. 2(a) and 2(b) show the performance of the MAS using SETF (3). We observe that approximate resilient consensus is guaranteed in this case, but triggering functions are frequently activated in the first few time steps. As a comparison, we next exhibit the performance of the DETF (5) in Figs. 3(a) and (b). Fig. 3(a) shows that the convergence rate of the DETF (5) is approximately equal to Fig. 2(a). From Fig. 3(b), it is observed that the communication times between agents are reduced in the first fifteen time steps. The detailed event counts are listed in TABLE I, from which we observe that with DETF (5), the number of triggered events for most cooperative agents has significantly decreased. These results mean that the proposed DE-MSR algorithm is superior in reducing communication overheads while maintaining a similar convergence to existing static event-triggered mechanisms.

V. CONCLUSION

In this brief, we address the resilient consensus problem for MASs in presence of malicious attacks by proposing a novel DE-MSR algorithm. Considering the existence of noncooperative agents in the network, the DE-MSR algorithm is executed to achieve resilient consensus with reduced communication overheads. A comparative case study is presented to exhibit the validity and superiority of the DE-MSR algorithm.

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