Resilient Consensus for Multi-Agent Systems (MASs) Through Dynamic Event-Triggered Mechanism

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Background

Swarm Performance

Agriculture

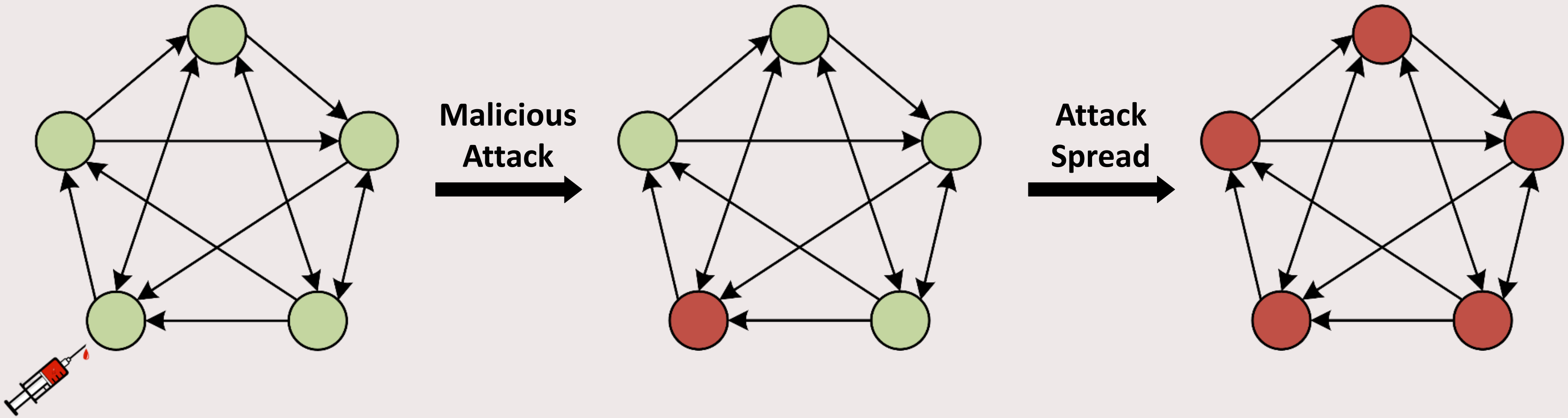
Application of MASs in Various Fields

Area Surveillance

Target Tracking

Background





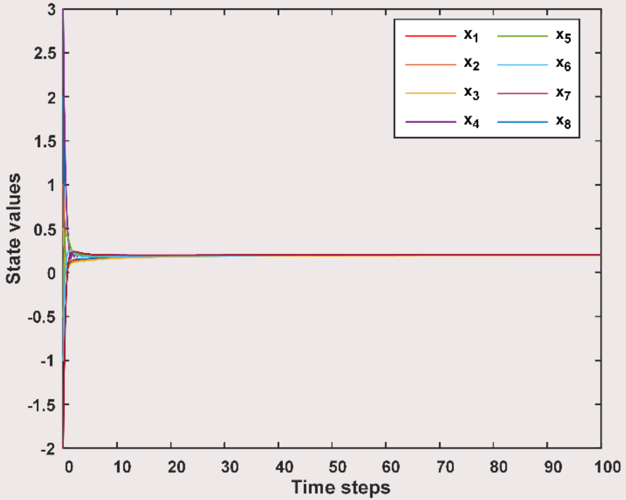
System Model

- Consider a single-integrator MAS modelled by

$$x_i[k+1] = x_i[k] + u_i[k]$$

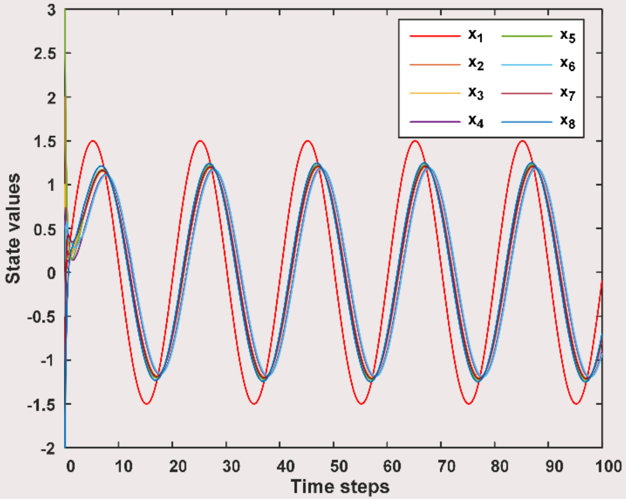
- A consensus-seeking protocol[3] is introduced as

$$u_i[k] = \sum_{j \in \mathcal{V}_i} a_{ij}[k]\left(x_j[k] - x_i[k]\right)$$



- Without malicious attack

Malicious Attack



- Agent 1 is subject to malicious attack

[3] W. Ren and Y. Cao, Distributed Coordination of Multi-Agent Networks: Emergent Problems, Models, and Issues. Springer, 2011.

Background

Definition 1 (Normal Agent). An agent is said to be normal if adopts the predetermined rule for state update.

Definition 2 (Malicious Agent). An agent is said to be malicious if it adopts some other function at some time step.

Challenges

- Resilient algorithm design
- Communication load
- Heterogeneity
- Time-varying networks
- ……

Existing Studies

Attack Detection and Isolation Method[1]

- Detecting and isolating malicious agents
- Agents in the network are equipped with observers
- Massive data processing
- High network connectivity required

Attack tolerant Method[2]

- Deleting malicious state values
- More lightweight with less computational complexity
- Less data processing
- Less prior information required

[1] I. Shames, A. M. Teixeira, H. Sandberg, and K. H. Johansson, "Distributed fault detection for interconnected second-order systems," Automatica, vol. 47, no. 12, pp. 2757–2764, 2011.
[2] H. J. LeBlanc, H. Zhang, X. Koutsoukos, and S. Sundaram, "Resilient asymptotic consensus in robust networks," IEEE J. Sel. Areas Commun., vol. 31, no. 4, pp. 766–781, 2013.

Resilient Consensus

- **Resilience**: For each normal agent $i \in \mathcal{N}$, it holds $x_i[k] \in \mathcal{I}, \quad \forall k \in \mathbb{Z}_{\geq 0}$, where

$$\mathcal{I} = \left[\min_{i \in \mathcal{N}} x_i[0], \max_{i \in \mathcal{N}} x_i[0]\right]$$

represents a safety interval.

- **(Exact) Consensus:** For each pair of normal agents $i, j \in \mathcal{N}$, it holds

$$\lim_{t \to \infty} \left| x_i[k] - x_j[k] \right| = 0, \quad \forall k \in \mathbb{Z}_{\geq 0}.$$

Mean Subsequence Reduced (MSR) Algorithm

1. Sorting

2. Deleting

3. Updating

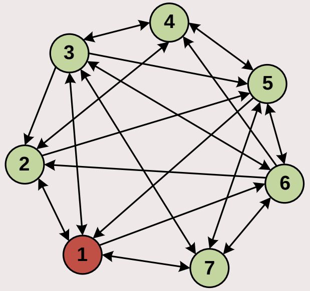
MSR Algorithm

1. Sorting

2. Deleting

3. Updating

Example:



$$[x_1[0], \cdots, x_7[0]] = [10, 8, 6, 7, 4, -5, 6], \;\; f = 2$$

Agent 2: $x_2[0] = 8 \quad x_j[0] = [10, 6, 7, -5]$

1. Sorting

$$S = [-5, 6, 7, 8, 10]^{\mathrm{T}}$$

2. Deleting

$$S_{del} = [\cancel{-5}, \cancel{6}, 7, 8, \cancel{10}]^{\mathrm{T}}, \quad \mathcal{R}_i[k] = \{4\}$$

3. Updating

$$x_i[k+1] = x_i[k] + \sum_{\boxed{j \in \mathcal{R}_i[k]}} a_{ij}[k]\big(x_j[k] - x_i[k]\big)$$

MSR Algorithm

1. Sorting
2. Deleting
3. Updating

Require $x_j[k]$ at each time step

Heavy communication load

Event-based MSR (E-MSR) Algorithm[4]

1. Sorting
2. Deleting
3. Updating
4. Checking

[4] Y. Wang and H. Ishii, "Resilient consensus through event-based communication," IEEE Trans. Control Netw. Syst., vol. 7, no. 1, pp. 471–482, 2020.

Static Event-Triggered Mechanism[4]

- The control protocol is modified as

$$x_i[k+1] = x_i[k] + \sum_{j \in \mathcal{R}_i[k]} a_{ij}[k]\big(\underbrace{\hat{x}_j[k]}_{\text{Auxiliary variable}} - x_i[k]\big)$$

- The update of auxiliary variable depends on

$$\hat{x}_i[k+1] = \begin{cases} x_i[k+1], & \text{if} \quad f_i[k] > 0, \\ \hat{x}_i[k], & \text{otherwise.} \end{cases}$$

- The static event-triggered function (SETF) is designed as

$$f_i[k] = \big|\underbrace{x_i[k+1] - \hat{x}_i[k]}_{\text{Error term } |e_i[k]|}\big| - \underbrace{\left(c_0 + c_1 e^{-\alpha k}\right)}_{\text{Threshold}}$$

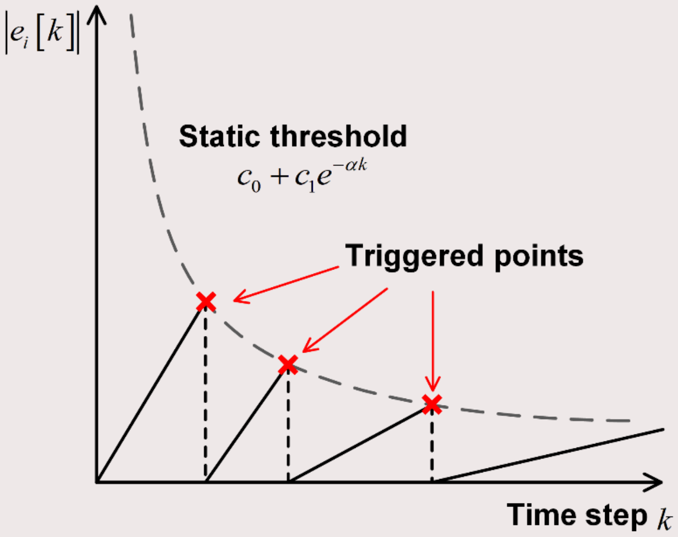


Illustration of the static event-triggered mechanism.

[4] Y. Wang and H. Ishii, “Resilient consensus through event-based communication,” IEEE Trans. Control Netw. Syst., vol. 7, no. 1, pp. 471–482, 2020.

E-MSR Algorithm[4]

1. Sorting

2. Deleting

3. Updating

4. Checking

4. Agent i checks whether SETF triggers or not and sets $\hat{x}_i[k+1]$ as

$$\hat{x}_i[k+1] = \begin{cases} x_i[k+1], & \text{if SETF triggers,} \\ \hat{x}_i[k], & \text{otherwise.} \end{cases}$$

Shortcomings:

- Only bounded resilient consensus can be achieved due to the existence of constant term c_0, i.e.,

$$\lim_{k\to\infty} \left| x_i[k] - x_j[k] \right| \le c, \quad \forall k \in \mathbb{Z}_{\ge 0}.$$

- The threshold cannot dynamically adjust as the error term changes.

[4] Y. Wang and H. Ishii, “Resilient consensus through event-based communication,” IEEE Trans. Control Netw. Syst., vol. 7, no. 1, pp. 471–482, 2020.

Dynamic Event-Triggered Mechanism

- The control protocol is also modified as

$$x_i[k+1] = x_i[k] + \sum_{j \in \mathcal{R}_i^{\text{in}}[k]} a_{ij}[k]\left(\hat{x}_j[k] - x_i[k]\right)$$

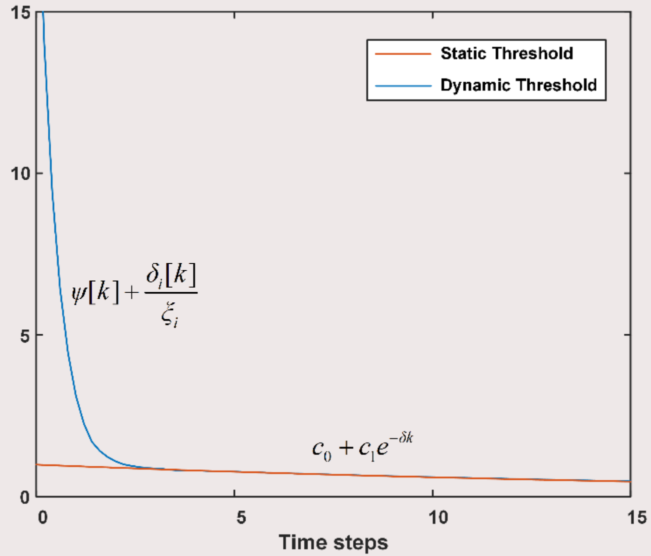
- A dynamic variable $\delta_i[k]$ is introduced, whose state update follows

$$\boxed{\delta_i[k+1] = (1-\theta_i)\delta_i[k] + \eta_i(\psi[k] - |e_i[k]|)}$$

Always greater than zero and converges to zero.

- The dynamic event-triggered function (DETF) is designed as

$$\boxed{f_i[k] = |e_i[k]| - \left(\psi[k] + \frac{\delta_i[k]}{\xi_i}\right)}$$



Comparison between the static and dynamic event-triggered mechanism.

Dynamic Event-triggered Mean Subsequence Reduced (DE-MSR) Algorithm[5]

1. Sorting

2. Deleting

3&5. Updating

4. Checking

4. Agent i checks whether DETF triggers or not and sets $\hat{x}_i[k+1]$ as

$$\hat{x}_i[k+1] = \begin{cases} x_i[k+1], & \text{if DETF triggers,} \\ \hat{x}_i[k], & \text{otherwise.} \end{cases}$$

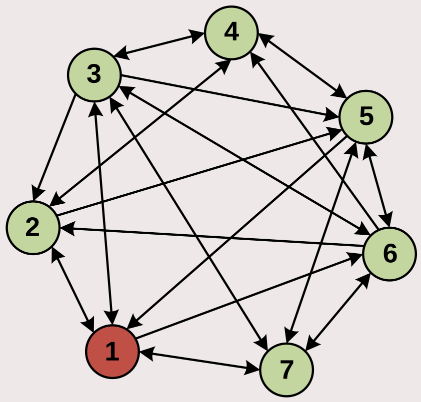
5. Agent i updates its interval dynamical variable according to

$$\delta_i[k+1] = (1-\theta_i)\delta_i[k] + \eta_i(\psi[k] - |e_i[k]|)$$

[5] **Z. Liao**, J. Shi, S. Wang, Y. Zhang, and Z. Sun, "Resilient consensus through dynamic event-triggered mechanism," IEEE Trans. Circuits Syst. II-Express Briefs, 2024, doi: 10.1109/TCSII.2024.3364524.

Comparative example between SETF and DETF

Communication topology



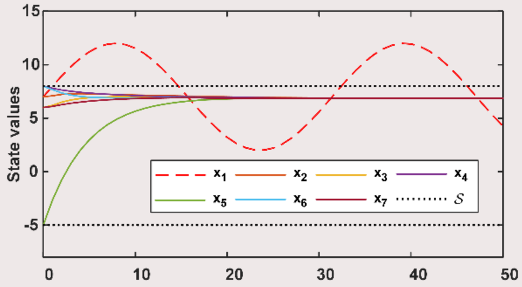
Simulation Setting

- Agent 1 is a malicious agent, whose motion follows
$$x_1[k] = 5 \times \sin(k/5) + 7$$
- Initial state values of all agents are denoted as
$$[x_1[0], \cdots, x_7[0]] = [7, 7, 6, 8, -5, 8, 6]$$
- SETF related parameters:
$$c_0 = 5 \times 10^{-4}, c_1 = 0.01, \alpha = 0.01$$
- DETF related parameters:
$$\xi_i = 10, \eta_i = 0.6, \theta_i = 0.35, \delta_i[0] = 15, \psi[k] = \mathrm{e}^{-\beta k}, \beta = 0.01$$

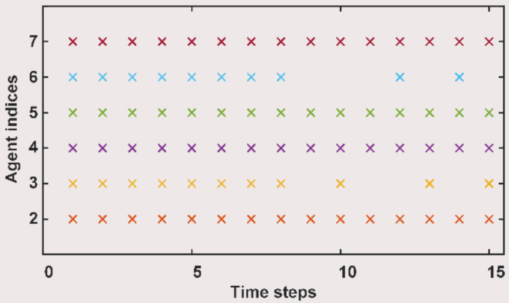
Purpose

- To validate the effectiveness of the proposed DE-MSR algorithm.
- To show the superiority of the DETF compared with the SETF.

Simulation Results

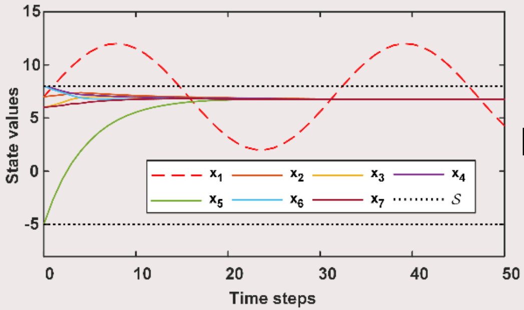


(a) Trajectory.

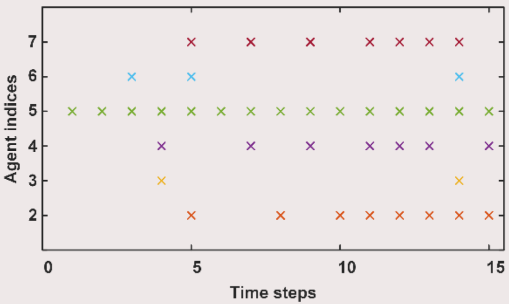


(b) Triggering behavior.

- SETF



(a) Trajectory.



(b) Triggering behavior.

- DETF

The SETF and DETF show a similar convergence rate.

The DETF is superior in reducing communication overheads.

Event-triggered mechanism	Event counts (within 250 time steps)					
	ag.2	ag.3	ag.4	ag.5	ag.6	ag.7
SETF	33	24	29	26	18	22
DETF	26	15	20	26	10	14

Conclusion

- Study the resilient consensus problem based on the dynamic event-triggered mechanism.
- Propose the DE-MSR algorithm to reduce communication overheads.
- Validate the effectiveness of the proposed method through numerical simulations.

Future Work

- MASs with disturbances generated from the external environment.
- More general and complex tasks (e.g., resilient distributed optimization).

Thanks for Listening!